从混合态构造相应的系统-热库纠缠纯态的新方法

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【摘要】 在量子统计学和低温物理学中,常用密度算符(或混合态)来表示物理系综.本文旨在提出一个新方法能将混合态转化为系统与热库相纠缠的量子纯态,这样做不但可以彰显环境对系统的影响,探寻新的物理态,尤其是纠缠态,而且能将求系综平均的复杂计算转化为较为简洁的求纯态平均.我们的新方法是将相干态和有序算符内的积分理论结合起来,就可直接将混合态扩充为自由度加倍的纯态.

关键词:系统-热库纠缠纯态,混合态,有序算符内的积分理论

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Constructing System-thermo Environment Pure Entangled State from a Mixed State

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【**Abstract**】 In quantum statistics theory and low temperature physics, we usually used are density operators (or mixed states). We propose a new approach for constructing the corresponding system-thermo environment pure entangled state from a mixed state. In this way we can not only exhibit the affection from the environment to the system, but also can find new entangled state and simplify the calculation of ensemble average to evaluating the pure state expectation value. Our new method lies in combining coherent state and the technique of integration within ordered product (IWOP) of operators. The pure entangled state is defined in real-fictitious two-mode space.

Keywords: System-thermo environment pure entangled state, Mixed state, IWOP method

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1 引 言

在量子统计学和低温物理学中[1-4]，常用密度算符(或混合态)来表示物理系综. 例如，最常见的频率为 ω 的混沌玻色场(声子场)的密度算符 ρ_c 是

$$\rho_c=(1-e^{\lambda})e^{\lambda a^{\dagger}a},\lambda=-\beta\hbar\omega,\beta=\frac{1}{kT} \tag{1}$$

k 是 Boltzmann 常数，T 为系统的温度，$a^{\dagger}a$ 是玻色子数算符，$[a,a^{\dagger}]=1$. 普朗克的玻色统计分布公式就可以从求 $tr(a^{\dagger}a\rho_c)$ 导出. 本文旨在提出一个新方法能将混合态转化为系统与热库相纠缠的量子纯态，以这样的方式引入热库，可以彰显环境对系统的影响，并能探寻与构建新的物理态，尤其是量子纠缠态，而且能将求系综平均的复杂计算转化为较为简洁的求纯态平均. 我们的新方法是将相干态和有序算符内的积分理论(IWOP)[5]结合起来，就可直接将混合态扩充为自由度扩充一倍的纯态.

2 混沌玻色场对应的系统-热库纠缠纯态

先简略介绍如何将相干态完备性纳入有序算符内的积分. 相干态 $|z\rangle$ 的定义是[6]

$$|z\rangle=\exp[-|z|^2/2+za^{\dagger}]|0\rangle \tag{2}$$

其完备性用真空投影算符的正规乘积形式

$$|0\rangle\langle 0|=:e^{-a^{\dagger}a}: \tag{3}$$

和 IWOP 方法就改写为

$$\begin{aligned}&\int\frac{d^2z}{\pi}|z\rangle\langle z|\\ &=\int\frac{d^2z}{\pi}:\exp[-|z|^2+za^{\dagger}+z^*a-a^{\dagger}a]:\\ &=\int\frac{d^2z}{\pi}:e^{-(z^*-a^{\dagger})(z-a)}:=1\end{aligned} \tag{4}$$

另一方面 ρ_c 的正规乘积展开是

$$\rho_c=(1-e^{\lambda}):\exp[(e^{\lambda}-1)a^{\dagger}a]: \tag{5}$$

记

$$e^{\lambda}=e^{-\beta\hbar\omega}\equiv\tanh^2\theta \tag{6}$$

于是就可以把混沌玻色场的密度算符 ρ_c 改写为

$$\begin{aligned}\rho_c&=\mathrm{sech}^2\theta:\exp[-\mathrm{sech}^2\theta a^{\dagger}a]:\\ &=\mathrm{sech}^2\theta\int\frac{d^2z}{\pi}:e^{-|z|^2+a^{\dagger}z^*\tanh\theta+az\tanh\theta-a^{\dagger}a}:\\ &=\mathrm{sech}^2\theta\int\frac{d^2z}{\pi}e^{a^{\dagger}z^*\tanh\theta}|0\rangle\langle 0|e^{az\tanh\theta}e^{-|z|^2}\end{aligned} \tag{7}$$

引入虚模相干态 $|\tilde{z}\rangle=\exp[-|z|^2/2+z\tilde{a}^{\dagger}]|\tilde{0}\rangle$，$|\tilde{0}\rangle$ 是虚模真空态，$\tilde{a}|\tilde{0}\rangle=0$，则由

$$e^{-|z|^2/2}=\langle\tilde{0}|\tilde{z}\rangle \tag{8}$$

和虚模相干态的完备性

$$\int\frac{d^2z}{\pi}|\tilde{z}\rangle\langle\tilde{z}|=1,\quad \tilde{a}|\tilde{z}\rangle=z|\tilde{z}\rangle \tag{9}$$

可将(7)式变成

$$\begin{aligned}\rho_c&=\mathrm{sech}^2\theta\int\frac{d^2z}{\pi}\langle\tilde{z}|e^{a^{\dagger}z^*\tanh\theta}|0,\tilde{0}\rangle\langle 0,\tilde{0}|e^{az\tanh\theta}|\tilde{z}\rangle\\ &=\mathrm{sech}^2\theta\int\frac{d^2z}{\pi}\langle\tilde{z}|e^{a^{\dagger}\tilde{a}^{\dagger}\tanh\theta}|0,\tilde{0}\rangle\langle 0,\tilde{0}|e^{a\tilde{a}\tanh\theta}|\tilde{z}\rangle\\ &=\mathrm{sech}^2\theta\tilde{tr}\left[\int\frac{d^2z}{\pi}e^{a^{\dagger}\tilde{a}^{\dagger}\tanh\theta}|0,\tilde{0}\rangle\langle 0,\tilde{0}|e^{a\tilde{a}\tanh\theta}|\tilde{z}\rangle\langle\tilde{z}|\right]\\ &=\mathrm{sech}^2\theta\tilde{tr}[e^{a^{\dagger}\tilde{a}^{\dagger}\tanh\theta}|0,\tilde{0}\rangle\langle 0,\tilde{0}|e^{a\tilde{a}\tanh\theta}]\end{aligned} \tag{10}$$

这里的 $\tilde{tr}$ 表示只是对虚模求迹. 令

$$|\psi(\beta)\rangle=\mathrm{sech}\theta e^{a^{\dagger}\tilde{a}^{\dagger}\tanh\theta}|0,\tilde{0}\rangle \tag{11}$$

上式化为

$$\begin{aligned}\rho_c&=\int\frac{d^2z}{\pi}\langle\tilde{z}|\psi(\beta)\rangle\langle\psi(\beta)\|\tilde{z}\rangle\\ &=\tilde{tr}\left[\int\frac{d^2z}{\pi}|\tilde{z}\rangle\langle\tilde{z}|\psi(\beta)\rangle\langle\psi(\beta)|\right]\\ &=\tilde{tr}|\psi(\beta)\rangle\langle\psi(\beta)|\end{aligned} \tag{12}$$

其中 $\tilde{tr}$ 是指对虚模求迹，$|\psi(\beta)\rangle$ 就是一个纯态，而且是个量子纠缠态，它将实模 $a^{\dagger}$ 和虚模 $\tilde{a}^{\dagger}$ 纠缠在一起，相当于系统-热库的纠缠态. 用这种方式我们可以自然地对给定系统引入热库，而且 $\langle\psi(\beta)|\psi(\beta)\rangle=1$.

3 纠缠纯态 $|\psi(\beta)\rangle$ 的应用

用纯态 $|\psi(\beta)\rangle$ 比较容易计算混沌场玻色子数平均值. 用

$$\begin{aligned}a|\psi(\beta)\rangle&=\mathrm{sech}\theta ae^{\tilde{a}^{\dagger}a^{\dagger}\tanh\theta}|0\tilde{0}\rangle\\ &=\mathrm{sech}\theta[a,e^{\tilde{a}^{\dagger}a^{\dagger}\tanh\theta}]|0\tilde{0}\rangle=\tilde{a}^{\dagger}\tanh\theta|\psi(\beta)\rangle\end{aligned} \tag{13}$$

计算

$$\langle\psi(\beta)\mid a^{\dagger}a\mid\psi(\beta)\rangle$$
$$=\operatorname{sech}^2\theta\langle 0\tilde{0}\mid e^{\tilde{a}a\tanh\theta}a^{\dagger}ae^{\tilde{a}^{\dagger}a^{\dagger}\tanh\theta}\mid 0\tilde{0}\rangle$$
$$=\tanh^2\theta[\langle\psi(\beta)\mid\tilde{a}^{\dagger}\tilde{a}\mid\psi(\beta)\rangle+1]\quad(14)$$

考虑到 $|\psi(\beta)\rangle$ 关于 $\tilde{a}^{\dagger}$ 与 $a^{\dagger}$ 是对称的,

$$\langle\psi(\beta)\mid a^{\dagger}a\mid\psi(\beta)\rangle=\langle\psi(\beta)\mid\tilde{a}^{\dagger}\tilde{a}\mid\psi(\beta)\rangle\equiv x\quad(15)$$

所以(14)式就是 $x=\tanh^2\theta(x+1)$,其解是

$$x=\sinh^2\theta=\langle\psi(\beta)\mid a^{\dagger}a\mid\psi(\beta)\rangle$$
$$=\frac{1}{1-e^{-\beta\hbar\omega}}e^{-\beta\hbar\omega}=(e^{-\beta\hbar\omega}-1)^{-1}\quad(16)$$

这就是普朗克玻色子分布公式,与计算 $tr(\rho_c a^{\dagger}a)$ 的结果相同.

我们再用 $|\psi(\beta)\rangle$ 计算混沌玻色场的熵,根据熵的表式 $S=kTr(\rho_c\ln\rho_c)$,利用 $|\psi(\beta)\rangle$ 后,熵 S 现在改为

$$S=k\langle\psi(\beta)\mid(\ln\rho_c)\mid\psi(\beta)\rangle$$
$$=k\langle\psi(\beta)\mid\ln[(1-e^{-\beta\hbar\omega})e^{-\beta\hbar\omega a^{\dagger}a}]\mid\psi(\beta)\rangle$$
$$=k\ln(1-e^{-\beta\hbar\omega})-k\beta\hbar\omega\langle\psi(\beta)\mid a^{\dagger}a\mid\psi(\beta)\rangle\quad(17)$$

直接代人(16)式的结果,立即得到

$$S=k\left[\ln(1-e^{-\beta\hbar\omega})+\frac{\beta\hbar\omega e^{-\beta\hbar\omega}}{e^{-\beta\hbar\omega}-1}\right]\quad(18)$$

可见,将混合态转换为系统-热库纠缠纯态的方式,不但提供了一种物理思路,而且处理一些问题是十分便利快捷的.

4 玻色场负二项式混合态对应的系统-热库纠缠纯态

考虑数理统计中的负二项分布 $\binom{n+s}{n}\gamma^{s+1}(1-\gamma)^n$,可以用 Fock 态 $|n\rangle$ 引入玻色场(声子场)的负二项分布态(混合态)[7]

$$\rho_s=\sum_{n=0}^{\infty}\frac{(n+s)!}{n!\,s!}\gamma^{s+1}(1-\gamma)^n\mid n\rangle\langle n\mid\quad(19)$$

用 $a\mid n\rangle=\sqrt{n}\mid n-1\rangle$ 容易证明

$$\rho_s=\gamma^{s+1}\frac{(1-\gamma)^{-s}}{s!}a^s\sum_{n=0}^{\infty}(1-\gamma)^n\mid n\rangle\langle n\mid a^{\dagger s}$$
$$=\frac{1}{s!\,n_c^s!}a^s\rho_c a^{\dagger s}\quad(20)$$

$$n_c=\frac{1}{\gamma}-1\quad(21)$$

ρ_c 即是(1)中的混沌玻色场

$$\rho_c=\sum_{n=0}^{\infty}\gamma(1-\gamma)^n\mid n\rangle\langle n\mid\overset{\gamma=1-e^f}{=}(1-e^f)e^{fa^{\dagger}a}\quad(22)$$

下面先将负二项式玻色场算符化为正规乘积形式. 利用 $|0\rangle\langle 0|=:e^{-a^{\dagger}a}:$ 和(22)式以及相干态的完备性关系,借助 IWOP 方法,可得

$$\rho_s=\frac{1}{s!\,n_c^s}a^s\rho_c a^{\dagger s}=\frac{1-e^f}{s!\,n_c^s}a^s e^{fa^{\dagger}a}a^{\dagger s}$$
$$=\frac{1-e^f}{s!\,n_c^s}\int\frac{d^2z}{\pi}a^s e^{fa^{\dagger}a}\mid z\rangle\langle z\mid a^{\dagger s}$$
$$=\frac{1-e^f}{s!\,n_c^s}\int\frac{d^2z}{\pi}e^{\frac{-|z|^2}{2}}a^s e^{fa^{\dagger}a}e^{za^{\dagger}}e^{-fa^{\dagger}a}\mid 0\rangle\langle z\mid z^{*s}$$
$$=\frac{1-e^f}{s!\,n_c^s}\int\frac{d^2z}{\pi}e^{\frac{-|z|^2}{2}}a^s e^{za^{\dagger}e^f}\mid 0\rangle\langle z\mid z^{*s}$$
$$=\frac{1-e^f}{s!\,n_c^s}\int\frac{d^2z}{\pi}(ze^f)^s z^{*s}:e^{-|z|^2+za^{\dagger}e^f+z^*a-a^{\dagger}a}:$$
$$=\frac{1-e^f}{s!\,n_c^s}e^{fs}:\sum_{l=0}^{\infty}e^{(e^f-1)a^{\dagger}a}\frac{(n!)^2(a^{\dagger}ae^f)^{n-l}}{l!\,[(n-l)!]^2}:$$
$$=(1-e^f)^{s+1}:e^{(e^f-1)a^{\dagger}a}L_s(-a^{\dagger}ae^f):$$
$$=\gamma^{s+1}:e^{-\gamma a^{\dagger}a}L_s[(\gamma-1)a^{\dagger}a]:\quad(23)$$

这里的 L_s 是 Laguerre 多项式,其定义是

$$L_s(x)=\sum_{l=0}^{s}\frac{(-x)^l n!}{(l!)^2(n-l)!}\quad(24)$$

利用式(23)以及 Laguerre 多项式的母函数公式

$$(1-z)^{-1}\exp\left[\frac{zx}{z-1}\right]=\sum_{l=0}^{\infty}L_n(x)z^n\quad(25)$$

不难验证

$$\frac{1-\gamma}{\gamma}\sum_{s=0}^{\infty}\rho_s$$
$$=(1-\gamma)\sum_{s=0}^{\infty}\gamma^s:e^{-\gamma a^{\dagger}a}L_s[(1-\gamma)a^{\dagger}a]:=1\quad(26)$$

这是负二项式混合态的正规乘积形式. 再求其所对应的系统-热库纠缠纯态,用有序算符内的积分方法,先考察 $a^s e^{\lambda a^{\dagger}a}a^{\dagger s}$ 即

$$a^s e^{\lambda a^{\dagger}a}a^{\dagger s}$$
$$=\int\frac{d^2z}{\pi}a^s:\exp(-|z|^2+z^*a^{\dagger}e^{\frac{\lambda}{2}}+zae^{\frac{\lambda}{2}}-a^{\dagger}a):a^{\dagger s}$$
$$=\int\frac{d^2z}{\pi}e^{-|z|^2}a^s\,\|z^*e^{\frac{\lambda}{2}}\rangle\langle ze^{\frac{\lambda}{2}}\|\,a^{\dagger s}\quad(27)$$

这里,$\|z^*e^{\lambda/2}\rangle$ 为未归一化的相干态. 注意到 $\langle\tilde{0}\mid\tilde{z}\rangle=e^{-|z|^2/2}$,将(27)式化为

$$
\begin{aligned}
a^s e^{\lambda a^\dagger a} a^{\dagger s} &= \int \frac{d^2 z}{\pi} z^s z^{*s} e^{\lambda s} e^{z^* a^\dagger e^{\lambda/2}} \mid 0\rangle\langle 0 \mid e^{z a e^{\lambda/2}} \langle \tilde{z} \mid \tilde{0}\rangle\langle \tilde{0} \mid \tilde{z}\rangle \\
&= \int \frac{d^2 z}{\pi} \langle \tilde{z} \mid z^s z^{*s} e^{\lambda s} e^{z^* a^\dagger e^{\lambda/2}} \mid 0,\tilde{0}\rangle\langle 0,\tilde{0} \mid e^{z a e^{\lambda/2}} \mid \tilde{z}\rangle \\
&= \int \frac{d^2 z}{\pi} \langle \tilde{z} \mid \tilde{a}^{\dagger s} e^{\lambda s} e^{\tilde{a}^\dagger a^\dagger e^{\lambda/2}} \mid 0,\tilde{0}\rangle\langle 0,\tilde{0} \mid e^{\tilde{a} a e^{\lambda/2}} \tilde{a}^s \mid \tilde{z}\rangle \\
&= e^{\lambda s} \tilde{tr} \left[\int \frac{d^2 z}{\pi} \tilde{a}^{\dagger s} e^{\tilde{a}^\dagger a^\dagger e^{\lambda/2}} \mid 0,\tilde{0}\rangle\langle 0,\tilde{0} \mid e^{\tilde{a} a e^{\lambda/2}} \tilde{a}^s \mid \tilde{z}\rangle\langle \tilde{z} \mid \right] \\
&= (1-\gamma)^s \tilde{tr} \left[\tilde{a}^{\dagger s} e^{\tilde{a}^\dagger a^\dagger e^{\lambda/2}} \mid 0,\tilde{0}\rangle\langle 0,\tilde{0} \mid e^{\tilde{a} a e^{\lambda/2}} \tilde{a}^s \right]
\end{aligned} \tag{28}
$$

代人(23)式得到

$$
\begin{aligned}
\rho_s &= \frac{\gamma}{s!\, n_c^s} a^s e^{\lambda a^\dagger a} a^{\dagger s} \\
&= \frac{\gamma^{s+1}}{s!} \tilde{tr} \left[\tilde{a}^{\dagger s} e^{\tilde{a}^\dagger a^\dagger e^{\lambda/2}} \mid 0,\tilde{0}\rangle\langle 0,\tilde{0} \mid e^{\tilde{a} a e^{\lambda/2}} \tilde{a}^s \right]
\end{aligned} \tag{29}
$$

对照(10)式可知相应于玻色场负二项式态的系统-热库纠缠纯态为

$$
\mid \psi(\beta) \rangle_s = \sqrt{\frac{\gamma^{s+1}}{s!}} \tilde{a}^{\dagger s} e^{\tilde{a}^\dagger a^\dagger \sqrt{1-\gamma}} \mid 0,\tilde{0}\rangle \tag{30}
$$

注意相对于 $\mid \psi(\beta) \rangle_s$ 而言,虚模激发 $\tilde{a}^{\dagger s}$ 等价于实模的湮灭,这与 $a^s e^{\lambda a^\dagger a} a^{\dagger s}$ 的表达式自洽;或者,将(30)改写为

$$
\begin{aligned}
\mid \psi(\beta) \rangle_s &= \sqrt{\gamma^{s+1}} \sum_{n=0}^{\infty} \frac{(\tilde{a}^\dagger a^\dagger \sqrt{1-\gamma})^n}{n!} \mid 0,\tilde{s}\rangle \\
&= \sum_{n=0}^{\infty} \sqrt{\binom{n+s}{n} \gamma^{s+1} (1-\gamma)^n} \mid n, \tilde{s}+\tilde{n}\rangle
\end{aligned} \tag{31}
$$

其中 $\mid n, \tilde{s}+\tilde{n}\rangle = \tilde{a}^{\dagger s+n} a^{\dagger n} \mid 0,\tilde{0}\rangle / \sqrt{n!\,(n+s)!}$. 对照式(19)式可见(31)中的的因子 $\sqrt{\binom{n+s}{n} \gamma^{s+1} (1-\gamma)^n}$ 恰好是负二项分布系数 $\binom{n+s}{n} \gamma^{s+1} (1-\gamma)^n$ 的开根号,所以 $\mid \psi(\beta) \rangle_s$ 确实是一个纠缠纯态. 而且可以用(30)式和相干态 $\tilde{a} \mid z_2\rangle = z_2 \mid z_2\rangle$ 直接证明:

$$
\begin{aligned}
{}_s\langle \psi(\beta) \mid \psi(\beta) \rangle_s &= \frac{\gamma^{s+1}}{s!} \langle \tilde{0},0 \mid \tilde{a}^s e^{\tilde{a} a \sqrt{1-\gamma}} \int \frac{d^2 z_1 d^2 z_2}{\pi^2} \mid z_1, z_2\rangle\langle z_1, z_2 \mid \tilde{a}^{\dagger s} e^{\tilde{a}^\dagger a^\dagger \sqrt{1-\gamma}} \mid 0,\tilde{0}\rangle \\
&= \frac{\gamma^{s+1}}{s!} \int \frac{d^2 z_1 d^2 z_2}{\pi^2} \mid z_2 \mid^{2s} e^{z_1 z_2 \sqrt{1-\gamma} + z_1^* z_2^* \sqrt{1-\gamma} - \mid z_2 \mid^2 - \mid z_1 \mid^2} = \frac{\gamma^{s+1}}{s!} \int \frac{d^2 z_2}{\pi} \mid z_2 \mid^{2s} e^{-\gamma \mid z_2 \mid^2} = 1
\end{aligned} \tag{32}
$$

即纯态 $\mid \psi(\beta) \rangle_s$ 是归一化的.

由(30)式给出

$$
\begin{aligned}
a \mid \psi(\beta) \rangle_s &= \sqrt{\frac{\gamma^{s+1}}{s!}} \sqrt{1-\gamma}\, \tilde{a}^{\dagger s+1} e^{\tilde{a}^\dagger a^\dagger \sqrt{1-\gamma}} \mid 0,\tilde{0}\rangle \\
&= \sqrt{1-\gamma} \sqrt{\frac{s+1}{\gamma}} \mid \psi(\beta) \rangle_{s+1}
\end{aligned} \tag{33}
$$

所以求 $a^\dagger a$ 的纯态平均立即可得玻色子数分布

$$
\begin{aligned}
&{}_s\langle \psi(\beta) \mid a^\dagger a \mid \psi(\beta) \rangle_s \\
&= (1-\gamma) \frac{s+1}{\gamma} {}_{s+1}\langle \psi(\beta) \mid \psi(\beta) \rangle_{s+1} \\
&= (1-\gamma) \frac{s+1}{\gamma} = (s+1) n_c
\end{aligned} \tag{34}
$$

注意到关系式

$$
\begin{aligned}
a^2 \mid \psi(\beta) \rangle_s &= \sqrt{\frac{\gamma^{s+1}}{s!}} (1-\gamma) \tilde{a}^{\dagger s+2} e^{\tilde{a}^\dagger \tilde{a} \sqrt{1-\gamma}} \mid 0,\tilde{0}\rangle \\
&= \frac{(1-\gamma)}{\gamma} \sqrt{(s+1)(s+2)} \mid \psi(\beta) \rangle_{s+2}
\end{aligned} \tag{35}
$$

和

$$
\begin{aligned}
{}_s\langle \psi(\beta) \mid a^{\dagger 2} a^2 \mid \psi(\beta) \rangle_s &= \frac{(1-\gamma)^2}{\gamma} (s+1)(s+2) \\
&= (s+1)(s+2) n_c^2
\end{aligned} \tag{36}
$$

可知处于负二项式纠缠纯态的玻色子数涨落为

$$
\begin{aligned}
&{}_s\langle \psi(\beta) \mid a^{\dagger 2} a^2 \mid \psi(\beta) \rangle_s - \left[{}_s\langle \psi(\beta) \mid a^\dagger a \mid \psi(\beta) \rangle_s \right]^2 \\
&= (s+1)(n_c+1) n_c
\end{aligned} \tag{37}
$$

负二项式纠缠态的二阶相干度为

$$
\frac{{}_s\langle \psi(\beta) \mid a^{\dagger 2} a^2 \mid \psi(\beta) \rangle_s}{\left[{}_s\langle \psi(\beta) \mid a^\dagger a \mid \psi(\beta) \rangle_s \right]^2} = \frac{s+2}{s+1} > 1 \tag{38}
$$

5 结　论

本文采用相干态和有序算符内的积分理论,给出了从混合态构造相应的系统-热库纠缠纯态的新方法,这丰富了有限温度下的量子统计理论.

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