

ON CONFORMABLE NABLA FRACTIONAL DERIVATIVE ON TIME SCALES

ZHAO Da-fang, YOU Xue-xiao, HU Chang-song

(*School of Mathematics and Statistics, Hubei Normal University, Huangshi 435002, China*)

Abstract: In this paper, we introduce and investigate the concept of conformable nabla fractional derivative on time scales. By using the theory of time scales, we obtain some basic properties of the conformable nabla fractional derivative, which extend and improve both the results in [9, 10] and the usual nabla derivative.

Keywords: conformable nabla fractional derivative; nabla derivative; time scales

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1 Introduction

Fractional Calculus is a generalization of ordinary differentiation and integration to arbitrary (non-integer) order. The subject is as old as the calculus of differentiation and goes back to times when Leibniz, Gauss, and Newton invented this kind of calculation. During three centuries, the theory of fractional calculus developed as a pure theoretical field, useful only for mathematicians. Nowadays, the fractional calculus attracts many scientists and engineers. There were several applications of this mathematical phenomenon in mechanics, physics, chemistry, control theory and so on [1–8].

Recently, the authors in [9] defined a new well-behaved simple fractional derivative called the conformable fractional derivative depending just on the basic limit definition of the derivative. Especially, in [10], Nadia Benkhettou, Salima Hassani and Delfim Torres introduced a conformable time-scale fractional derivative, which providing a natural extension of the conformable fractional derivative. In this paper, we define the conformable nabla fractional derivative on time scales, which give another type of generalization of the conformable fractional derivative and the usual nabla derivative [11–14].

2 Preliminaries

A time scale \mathbb{T} is a nonempty closed subset of real numbers \mathbb{R} with the subspace topology inherited from the standard topology of \mathbb{R} . For $a, b \in \mathbb{T}$ we define the closed interval $[a, b]_{\mathbb{T}}$

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by $[a, b]_{\mathbb{T}} = \{t \in \mathbb{T} : a \leq t \leq b\}$. For $t \in \mathbb{T}$ we define the forward jump operator σ by $\sigma(t) = \inf\{s > t : s \in \mathbb{T}\}$, where $\inf \emptyset = \sup \mathbb{T}$, while the backward jump operator ρ is defined by $\rho(t) = \sup\{s < t : s \in \mathbb{T}\}$, where $\sup \emptyset = \inf \mathbb{T}$.

If $\sigma(t) > t$, we say that t is right-scattered, while if $\rho(t) < t$, we say that t is left-scattered. If $\sigma(t) = t$, we say that t is right-dense, while if $\rho(t) = t$, we say that t is left-dense. A point $t \in \mathbb{T}$ is dense if it is right and left dense; isolated if it is right and left scattered. The forward graininess function $\mu(t)$ and the backward graininess function $\eta(t)$ are defined by $\mu(t) = \sigma(t) - t$, $\eta(t) = t - \rho(t)$ for all $t \in \mathbb{T}$, respectively. If $\sup \mathbb{T}$ is finite and left-scattered, then we define $\mathbb{T}^k := \mathbb{T} \setminus \sup \mathbb{T}$, otherwise $\mathbb{T}^k := \mathbb{T}$; if $\inf \mathbb{T}$ is finite and right-scattered, then $\mathbb{T}_k := \mathbb{T} \setminus \inf \mathbb{T}$, otherwise $\mathbb{T}_k := \mathbb{T}$. We set $\mathbb{T}_k^k := \mathbb{T}^k \cap \mathbb{T}_k$.

A function $f : \mathbb{T} \rightarrow \mathbb{R}$ is nabla (∇) differentiable at $t \in \mathbb{T}_k$ if there exists a number $f^\nabla(t)$ such that, for each $\varepsilon > 0$, there exists a neighborhood U of t such that

$$|f(\rho(t)) - f(s) - f^\nabla(t)(\rho(t) - s)| \leq \varepsilon|\rho(t) - s|$$

for all $s \in U$. We call $f^\nabla(t)$ the ∇ -derivative of f at t . Throughout this paper, $\alpha \in (0, 1]$.

3 Conformable Nabla Fractional Derivative

Definition 3.1 Let \mathbb{T} be a time scale and $\alpha \in (0, 1]$. A function $f : \mathbb{T} \rightarrow \mathbb{R}$ is conformable ∇ -fractional differentiable of order α at $t \in \mathbb{T}_k$ if there exists a number $\mathbf{T}_\alpha(f^\nabla)(t)$ such that, for each $\varepsilon > 0$, there exists a neighborhood U of t such that

$$|(f(\rho(t)) - f(s))\rho(t)^{1-\alpha} - \mathbf{T}_\alpha(f^\nabla)(t)(\rho(t) - s)| \leq \varepsilon|\rho(t) - s|$$

for all $s \in U$. We call $\mathbf{T}_\alpha(f^\nabla)(t)$ the conformable ∇ -fractional derivative of f of order α at t and we say that f is conformable ∇ -fractional differentiable if f is conformable ∇ -fractional differentiable for all $t \in \mathbb{T}_k$.

Theorem 3.2 Let \mathbb{T} be a time scale, $t \in \mathbb{T}_k$ and $\alpha \in (0, 1]$. Then we have the following:

(i) If f is conformal ∇ -fractional differentiable of order α at t , then f is continuous at t .

(ii) If f is continuous at t and t is left-scattered, then f is conformable ∇ -fractional differentiable of order α at t with $\mathbf{T}_\alpha(f^\nabla)(t) = -\frac{f(\rho(t)) - f(t)}{\eta(t)}\rho(t)^{1-\alpha}$.

(iii) If t is left-dense, then f is conformable ∇ -fractional differentiable of order α at t if and only if the limit $\lim_{s \rightarrow t} \frac{f(t) - f(s)}{t - s}t^{1-\alpha}$ exists as a finite number. In this case,

$$\mathbf{T}_\alpha(f^\nabla)(t) = \lim_{s \rightarrow t} \frac{f(t) - f(s)}{t - s}t^{1-\alpha}.$$

(iv) If f is conformal ∇ -fractional differentiable of order α at t , then

$$f(\rho(t)) = f(t) - \eta(t)\mathbf{T}_\alpha(f^\nabla)(t)\rho(t)^{\alpha-1}.$$

Proof (i) The proof is easy and will be omitted.

(ii) Assume that f is continuous at t and t is left-scattered. By continuity,

$$\lim_{s \rightarrow t} \frac{f(\rho(t)) - f(s)}{\rho(t) - s} \rho(t)^{1-\alpha} = \frac{f(\rho(t)) - f(t)}{\rho(t) - t} \rho(t)^{1-\alpha} = \frac{f(\rho(t)) - f(t)}{-\eta(t)} \rho(t)^{1-\alpha}.$$

Hence given $\varepsilon > 0$, there exists a neighborhood U of t such that

$$\left| \frac{f(\rho(t)) - f(s)}{\rho(t) - s} \rho(t)^{1-\alpha} - \frac{f(\rho(t)) - f(s)}{-\eta(t)} \rho(t)^{1-\alpha} \right| \leq \varepsilon$$

for all $s \in U$. It follows that

$$\left| (f(\rho(t)) - f(s)) \rho(t)^{1-\alpha} - \frac{f(\rho(t)) - f(s)}{-\eta(t)} (\rho(t) - s) \rho(t)^{1-\alpha} \right| \leq \varepsilon |\rho(t) - s|$$

for all $s \in U$. Hence we get the desired result $\mathbf{T}_\alpha(f^\nabla)(t) = -\frac{f(\rho(t)) - f(t)}{\eta(t)} \rho(t)^{1-\alpha}$.

(iii) Assume that f is conformable ∇ -fractional differentiable of order α at t and t is right-dense. Then for each $\varepsilon > 0$, there exists a neighborhood U of t such that

$$|(f(\rho(t)) - f(s)) \rho(t)^{1-\alpha} - \mathbf{T}_\alpha(f^\nabla)(t)(\rho(t) - s)| \leq \varepsilon |\rho(t) - s|$$

for all $s \in U$. Since $\rho(t) = t$ we have that $|(f(t) - f(s)) t^{1-\alpha} - \mathbf{T}_\alpha(f^\nabla)(t)(t - s)| \leq \varepsilon |t - s|$ for all $s \in U$. It follows that $\left| \frac{f(t) - f(s)}{t - s} t^{1-\alpha} - \mathbf{T}_\alpha(f^\nabla)(t) \right| \leq \varepsilon$ for all $s \in U$, $s \neq t$. Hence we get the desired result.

On the other hand, if the limit $\lim_{s \rightarrow t} \frac{f(t) - f(s)}{t - s} t^{1-\alpha}$ exists as a finite number and is equal to J , then for each $\varepsilon > 0$, there exists a neighborhood U of t such that

$$|(f(t) - f(s)) t^{1-\alpha} - J(t - s)| \leq \varepsilon |t - s|$$

for all $s \in U$. Since t is right-dense, we have that

$$|(f(\rho(t)) - f(s)) \rho(t)^{1-\alpha} - J(\rho(t) - s)| \leq \varepsilon |\rho(t) - s|.$$

Hence f is conformable ∇ -fractional differentiable at t and $\mathbf{T}_\alpha(f^\nabla)(t) = \lim_{s \rightarrow t} \frac{f(t) - f(s)}{t - s} t^{1-\alpha}$.

(iv) If t is left-dense, then $\eta(t) = 0$ and we have that

$$f(\rho(t)) = f(t) = f(t) + \eta(t) \mathbf{T}_\alpha(f^\nabla)(t) \rho(t)^{\alpha-1}.$$

If t is left-scattered, then $\rho(t) < t$, then by (ii)

$$f(\rho(t)) = f(t) + \eta(t) \frac{f(\rho(t)) - f(t)}{\eta(t)} = f(t) - \eta(t) \mathbf{T}_\alpha(f^\nabla)(t) \rho(t)^{\alpha-1}.$$

Corollary 3.3 Again we consider the two cases $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$.

(i) If $\mathbb{T} = \mathbb{R}$, then $f : \mathbb{R} \rightarrow \mathbb{R}$ is conformable ∇ -fractional differentiable of order α at $t \in \mathbb{R}$ if and only if the limit $\lim_{s \rightarrow t} \frac{f(t) - f(s)}{t - s} t^{1-\alpha}$ exists as a finite number. In this case,

$$\mathbf{T}_\alpha(f^\nabla)(t) = \lim_{s \rightarrow t} \frac{f(t) - f(s)}{t - s} t^{1-\alpha}.$$

If $\alpha = 1$, then we have that $\mathbf{T}_\alpha(f^\nabla)(t) = f^\nabla(t) = f'(t)$.

(ii) if $\mathbb{T} = \mathbb{Z}$, then $f : \mathbb{Z} \rightarrow \mathbb{R}$ is conformable ∇ -fractional differentiable of order α at $t \in \mathbb{Z}$ with

$$\mathbf{T}_\alpha(f^\nabla)(t) = -\frac{f(t-1) - f(t)}{1}(t-1)^{1-\alpha} = (t-1)^{1-\alpha}(f(t) - f(t-1)).$$

If $\alpha = 1$, then we have that $\mathbf{T}_\alpha(f^\nabla)(t) = f(t) - f(t-1) = \nabla f(t)$, where ∇ is the usual backward difference operator.

Example 3.4 If $f : \mathbb{T} \rightarrow \mathbb{R}$ is defined by $f(t) = C$ for all $t \in \mathbb{T}$, where $C \in \mathbb{R}$ is constant, then $\mathbf{T}_\alpha(f^\nabla)(t) \equiv 0$.

(ii) if $f : \mathbb{T} \rightarrow \mathbb{R}$ is defined by $f(t) = t$ for all $t \in \mathbb{T}$, then $\mathbf{T}_\alpha(f^\nabla)(t) = \rho(t)^{1-\alpha}$. If $\alpha = 1$, then $\mathbf{T}_\alpha(f^\nabla)(t) \equiv 1$.

Example 3.5 If $f : \mathbb{T} \rightarrow \mathbb{R}$ is defined by $f(t) = t^2$ for all $t \in \mathbb{T} := \{\frac{n}{2} : n \in \mathbb{N}_0\}$, then from Theorem 3.2 (ii) we have that f is conformable ∇ -fractional differentiable of order α at $t \in \mathbb{T}$ with

$$\mathbf{T}_\alpha(f^\nabla)(t) = \left(2t - \frac{1}{2}\right) \left(t - \frac{1}{2}\right)^{1-\alpha}.$$

Theorem 3.6 Assume $f, g : \mathbb{T} \rightarrow \mathbb{R}$ are conformable ∇ -fractional differentiable of order α at $t \in \mathbb{T}_k$, then

(i) for any constant λ_1, λ_2 , the sum $\lambda_1 f + \lambda_2 g : \mathbb{T} \rightarrow \mathbb{R}$ is conformable ∇ -fractional differentiable of order α at t with $\mathbf{T}_\alpha((\lambda_1 f + \lambda_2 g)^\nabla)(t) = \lambda_1 \mathbf{T}_\alpha(f^\nabla)(t) + \lambda_2 \mathbf{T}_\alpha(g^\nabla)(t)$;

(ii) if f and g are continuous, then the product $fg : \mathbb{T} \rightarrow \mathbb{R}$ is conformable ∇ -fractional differentiable of order α at t with

$$\mathbf{T}_\alpha(fg)^\nabla(t) = \mathbf{T}_\alpha(f^\nabla)(t)g(t) + f(\rho(t))\mathbf{T}_\alpha(g^\nabla)(t) = f(t)\mathbf{T}_\alpha(g^\nabla)(t) + \mathbf{T}_\alpha(f^\nabla)(t)g(\rho(t));$$

(iii) if $f(t)f(\rho(t)) \neq 0$, then $\frac{1}{f}$ is conformable ∇ -fractional differentiable of order α at t with

$$\mathbf{T}_\alpha\left(\frac{1}{f}\right)^\nabla(t) = -\frac{\mathbf{T}_\alpha(f^\nabla)(t)}{f(t)f(\rho(t))};$$

(iv) if $g(t)g(\rho(t)) \neq 0$, then $\frac{f}{g}$ is conformable ∇ -fractional differentiable of order α at t with

$$\mathbf{T}_\alpha\left(\frac{f}{g}\right)^\nabla(t) = \frac{\mathbf{T}_\alpha(f^\nabla)(t)g(t) - f(t)\mathbf{T}_\alpha(g^\nabla)(t)}{g(t)g(\rho(t))}.$$

Proof (i) The proof is easy and will be omitted.

(ii) Let $0 < \varepsilon < 1$. Define

$$\epsilon^* = \frac{\epsilon}{1 + |g(\rho(t))| + |f(t)| + |\mathbf{T}_\alpha(g^\nabla)(t)|},$$

then $0 < \epsilon^* < 1$. $f, g : \mathbb{T} \rightarrow \mathbb{R}$ are conformable ∇ -fractional differentiable of order α at t . Then there exists neighborhoods U_1 and U_2 of t with

$$|(f(\rho(t)) - f(s))\rho(t)^{1-\alpha} - \mathbf{T}_\alpha(f^\nabla)(t)(\rho(t) - s)| \leq \epsilon^*|\rho(t) - s|$$

for all $s \in U_1$ and

$$|g(\rho(t)) - g(s))\rho(t)^{1-\alpha} - \mathbf{T}_\alpha(g^\nabla)(t)(\rho(t) - s)| \leq \epsilon^* |\rho(t) - s|$$

for all $s \in U_2$.

From Theorem 3.2 (i), there exists neighborhoods U_3 of t with $|f(t) - f(s)| \leq \epsilon^*$ for all $s \in U_3$.

Let $U = U_1 \cap U_2 \cap U_3$. Then we have for all $s \in U$

$$\begin{aligned} & |[f(\rho(t))g(\rho(t)) - f(s)g(s)]\rho(t)^{1-\alpha} - [\mathbf{T}_\alpha(f^\nabla)(t)g(\rho(t)) + f(t)\mathbf{T}_\alpha(g^\nabla)(t)](\rho(t) - s)| \\ \leq & |[f(\rho(t)) - f(s)]\rho(t)^{1-\alpha} - \mathbf{T}_\alpha(f^\nabla)(t)(\rho(t) - s)]g(\rho(t))| \\ & + |[g(\rho(t)) - g(s)]\rho(t)^{1-\alpha} - \mathbf{T}_\alpha(g^\nabla)(t)(\rho(t) - s)]f(t)| \\ & + |[g(\rho(t)) - g(s)]\rho(t)^{1-\alpha} - \mathbf{T}_\alpha(g^\nabla)(t)(\rho(t) - s)](f(s) - f(t))| \\ & + |\mathbf{T}_\alpha(g^\nabla)(t)(\rho(t) - s)](f(s) - f(t))| \\ \leq & \epsilon^* |\rho(t) - s| \cdot (|g(\rho(t))| + |f(t)| + \epsilon^* + |\mathbf{T}_\alpha(g^\nabla)(t)|) \\ \leq & \epsilon |\rho(t) - s|. \end{aligned}$$

Thus $\mathbf{T}_\alpha(fg)^\nabla(t) = f(t)\mathbf{T}_\alpha(g^\nabla)(t) + \mathbf{T}_\alpha(f^\nabla)(t)g(\rho(t))$. The other product rule formula follows by interchanging the role of functions f and g .

(iii) From Example 3.4, we have that $\mathbf{T}_\alpha\left(f \cdot \frac{1}{f}\right)^\nabla(t) = \mathbf{T}_\alpha(1)^\nabla(t) = 0$. Therefore

$$\mathbf{T}_\alpha\left(\frac{1}{f}\right)^\nabla(t)f(\rho(t)) + \mathbf{T}_\alpha(f^\nabla)(t)\frac{1}{f(t)} = 0$$

and consequently $\mathbf{T}_\alpha\left(\frac{1}{f}\right)^\nabla(t) = -\frac{\mathbf{T}_\alpha(f^\nabla)(t)}{f(t)f(\rho(t))}$.

(iv) We use (ii) and (iii) to calculate

$$\begin{aligned} \mathbf{T}_\alpha\left(\frac{f}{g}\right)^\nabla(t) &= f(t)\mathbf{T}_\alpha\left(\frac{1}{g}\right)^\nabla(t) + \mathbf{T}_\alpha(f^\nabla)(t)\frac{1}{g(\rho(t))} \\ &= -f(t)\frac{\mathbf{T}_\alpha(g^\nabla)(t)}{g(t)g(\rho(t))} + \mathbf{T}_\alpha(f^\nabla)(t)\frac{1}{g(\rho(t))} \\ &= \frac{\mathbf{T}_\alpha(f^\nabla)(t)g(t) - f(t)\mathbf{T}_\alpha(g^\nabla)(t)}{g(t)g(\rho(t))}. \end{aligned}$$

Theorem 3.7 Let c be constant and $m \in \mathbb{N}$.

(i) For f defined by $f(t) = (t - c)^m$, we have that

$$\mathbf{T}_\alpha(f^\nabla)(t) = \rho(t)^{1-\alpha} \sum_{i=0}^{m-1} (\rho(t) - c)^i (t - c)^{m-1-i}.$$

(ii) For g defined by $g(t) = \frac{1}{(t-c)^m}$, we have that

$$\mathbf{T}_\alpha(g^\nabla)(t) = -\rho(t)^{1-\alpha} \sum_{i=0}^{m-1} \frac{1}{(\rho(t) - c)^{m-i}(t - c)^{i+1}}$$

provided $(\rho(t) - c)(t - c) \neq 0$.

Proof (i) We prove the first formula by induction. If $m = 1$, then $f(t) = t - c$, and clearly $\mathbf{T}_\alpha(f^\nabla)(t) = \rho(t)^{1-\alpha}$ holds by Example 3.4 and Theorem 3.6(i). Now we assume that

$$\mathbf{T}_\alpha(f^\nabla)(t) = \rho(t)^{1-\alpha} \sum_{i=0}^{m-1} (\rho(t) - c)^i (t - c)^{m-1-i}$$

holds for $f(t) = (t - c)^m$ and let $F(t) = (t - c)^{m+1} = (t - c)f(t)$. We use Theorem 3.6 (ii) to obtain

$$\begin{aligned} \mathbf{T}_\alpha(F)^\nabla(t) &= \rho(t)^{1-\alpha} f(\rho(t)) + (t - c)\mathbf{T}_\alpha(f^\nabla)(t) \\ &= \rho(t)^{1-\alpha} (\rho(t) - c)^m + (t - c)\rho(t)^{1-\alpha} \sum_{i=0}^{m-1} (\rho(t) - c)^i (t - c)^{m-1-i} \\ &= \rho(t)^{1-\alpha} \sum_{i=0}^m (\rho(t) - c)^i (t - c)^{m-i}. \end{aligned}$$

Hence part (i) holds.

(ii) For $g(t) = \frac{1}{(t-c)^m}$, we use Theorem 3.6 (iii) to obtain

$$\begin{aligned} \mathbf{T}_\alpha(g)^\nabla(t) &= -\frac{\mathbf{T}_\alpha(f^\nabla)(t)}{f(t)f(\rho(t))} \\ &= -\frac{\rho(t)^{1-\alpha} \sum_{i=0}^{m-1} (\rho(t) - c)^i (t - c)^{m-1-i}}{(\rho(t) - c)^m (t - c)^m} \\ &= -\rho(t)^{1-\alpha} \sum_{i=0}^{m-1} \frac{1}{(\rho(t) - c)^{m-i} (t - c)^{i+1}} \end{aligned}$$

provided $(\rho(t) - c)(t - c) \neq 0$.

Example 3.8 If $f : \mathbb{T} \rightarrow \mathbb{R}$ is defined by $f(t) = \frac{1}{t^2}$ for all $t \in \mathbb{T} := \{\sqrt{n} : n \in \mathbb{N}_0\}$, then we have that f is conformable ∇ -fractional differentiable of order α at $t \in \mathbb{T}$ with

$$\mathbf{T}_\alpha(f^\nabla)(t) = -\rho(t)^{1-\alpha} \left(\frac{1}{(\rho(t))^{2t}} + \frac{1}{\rho(t)t^2} \right) = -(\sqrt{t^2 - 1})^{-\alpha} \left(\frac{1}{t\sqrt{t^2 - 1}} + \frac{1}{t^2} \right).$$

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关于时标上的适应Nabla 分数阶导数

赵大方, 游雪肖, 胡长松

(湖北师范大学数学与统计学院, 湖北 黄石 435002)

摘要: 本文研究了时标上的适应Nabla 分数阶导数的问题. 利用时标理论, 获得了关于适应Nabla 分数阶导数的若干重要性质. 这些结果推广并改进了文献 [9, 10] 中的有关结论以及一般Nabla 导数的性质.

关键词: 适应Nabla 分数阶导数; Nabla 导数; 时标

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