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THE L^2 EIGENFORMS OF THE LAPLACIAN ON COMPLETE MANIFOLDS

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Abstract: In this paper, we investigate the L^2 eigenforms of the Laplacian on complete noncompact manifolds. By using the method of stress energy tensor, we obtain some nonexistence theorems of L^2 eigenforms of the Laplacian on these manifolds.

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1 Introduction

In this paper, we consider the nonexistence of L^2 eigenforms of the Laplacian on a complete noncompact manifold M under various conditions, such as having exhaustion functions which satisfy some special conditions or having various pinching radial sectional curvature. It is well known that the Hodge Laplacian $\triangle^p = d\delta + \delta d$ is self-adjoint on $L^2A^p(M)$, indeed, essentially self-adjoint on the space $C_0^\infty A^p(M)$ of compactly supported smooth p-forms [1]. We denote the corresponding operator domain with the symbol $\mathrm{dom}(\triangle^p)$.

Our main results are first based on some Rellich-type identities for differential forms, analogous to those obtained by [2] and [3], but we find it natural and direct to express them by stress energy tensors. Then we specialize the discussion to the case that the metric g is the conformal deformation of the background metric g_0 and obtain the corresponding integral formula. By this integral formula, we can obtain conditions on the conformal function f and on other geometric conditions under which $\triangle_g^p$ has no positive point spectrum, i.e., there are no nonzero square integrable p-form u in $\mathrm{dom}(\triangle^p)$ satisfying the eigenvalue equation $\triangle^p u = \lambda u$ $(\lambda > 0)$. The main feature of these results is that in all cases we allow a controlled conformal deformation of the metric. Our results improve and complement those obtained by [2–4].

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We consider the nonexistence of eigenforms under the conditions of exhaustion function whose Hessian satisfies some pinching conditions. When we choose the special exhaustion function to be the square of the distance function $r(x)$, where $x \in M$, we can relate these conditions to the radial sectional curvature of the complete manifold with a pole by Hessian comparison theorem, and obtain nonexistence theorems under various pinching condition on radial sectional curvature.

2 Stress Energy Tensors and Exhaustion Function

Let (M, g) be a Riemannian manifold and $\xi : E \to M$ be a smooth Riemannian vector bundle over M with a metric compatible connection ∇^E. Set $A^p(\xi) = \Gamma(\Lambda^p T^*M \otimes E)$ the space of smooth p-forms on M with values in the vector bundle $\xi : E \to M$. When ξ is the trivial bundle $M \times R$, denote $A^p(M) = \Gamma(\Lambda^p T^*M)$. The exterior covariant differentiation $d^\nabla : A^p(\xi) \to A^{p+1}(\xi)$ relative to the connection ∇^E is defined by

$$(d^\nabla \omega)(X_1, \cdots, X_{p+1}) = \sum_{i=1}^{p+1} (-1)^{i+1} (\nabla_{X_i}\omega)(X_1, \cdots, \widehat{X_i}, \cdots, X_{p+1}).$$

The codifferential operator $\delta^\nabla : A^p(\xi) \to A^{p-1}(\xi)$ characterized as the adjoint of d^∇ is defined by

$$(\delta^\nabla \omega)(X_1, \cdots, X_{p-1}) = -\sum_i (\nabla_{e_i}\omega)(e_i, X_1, \cdots, X_{p-1}).$$

Given ω and θ in $A^p(\xi)$, the induced inner product on $\wedge^p T^*_x M \otimes E_x$ is defined as follows:

$$\begin{aligned} \langle \omega, \theta \rangle &= \sum_{i_1 < \cdots < i_p} \langle \omega(e_{i_1}, \cdots, e_{i_p}), \theta(e_{i_1}, \cdots, e_{i_p}) \rangle_{E_x} \\ &= \frac{1}{p!} \sum_{i_1, \cdots, i_p} \langle \omega(e_{i_1}, \cdots, e_{i_p}), \theta(e_{i_1}, \cdots, e_{i_p}) \rangle_{E_x} \end{aligned}$$

and denote by $|\cdot|$ the induced norm. The energy functional of $\omega \in A^p(\xi)$ is defined to be

$$E(\omega) = \frac{1}{2} \int_M |\omega|^2 dv_g,$$

its stress-energy tensor is

$$S_\omega(X, Y) = \frac{|\omega|^2}{2} g(X, Y) - (\omega \odot \omega)(X, Y), \tag{2.1}$$

where $\omega \odot \omega \in \Gamma(A^p(\xi) \otimes A^p(\xi))$ is a symmetric tensor defined by

$$(\omega \odot \omega)(X, Y) = \langle i_X \omega, i_Y \omega \rangle,$$

here $i_X \omega \in A^{p-1}(\xi)$ denotes the interior product by $X \in TM$. Notice that, if $p = 0$, i.e., $\omega \in \Gamma(\xi)$, $i_X \omega = 0$ then (2.1) becomes

$$S_\omega(X, Y) = \frac{|\omega|^2}{2} g(X, Y). \tag{2.2}$$

For a 2-tensor field $T \in \Gamma(T^*M \otimes T^*M)$, its divergence $\mathrm{div}T \in \Gamma(T^*M)$ is defined by

$$(\mathrm{div}T)(X) = \sum_i (\nabla_{e_i} T)(e_i, X),$$

where $\{e_i\}$ is an orthonormal basis of TM. The divergence of S_ω is given by (see [5–7])

$$(\mathrm{div}S_\omega)(X) = \langle \delta^\nabla \omega, i_X \omega \rangle + \langle i_X d^\nabla \omega, \omega \rangle. \tag{2.3}$$

For a vector field X on M, its dual one form θ_X is given by

$$\theta_X(Y) = g(X, Y), \forall Y \in TM.$$

The covariant derivative of θ_X gives a 2-tensor field $\nabla \theta_X$:

$$(\nabla \theta_X)(Y, Z) = (\nabla_Z \theta_X)(Y) = g(\nabla_Z X, Y), \forall Y, Z \in TM. \tag{2.4}$$

If $X = \nabla \psi$ is the gradient of some smooth function ψ on M, then $\theta_X = d\psi$ and $\nabla \theta_X = \mathrm{Hess}(\psi)$.

For any vector field X on M, a direct computation yields (see [6] or Lemma 2.4 of [3])

$$\mathrm{div}(i_X S_\omega) = \langle S_\omega, \nabla \theta_X \rangle + (\mathrm{div}S_\omega)(X). \tag{2.5}$$

Let D be any bounded domain of M with C^1 boundary. By (2.5) and using the divergence theorem, we immediately have the following integral formula (see [6, 8])

$$\int_{\partial D} S_\omega(X, \nu) dv_{\partial D} = \int_D [\langle S_\omega, \nabla \theta_X \rangle + (\mathrm{div}S_\omega)(X)] dv_g, \tag{2.6}$$

where ν is the unit outward normal vector field along ∂D.

To apply the above integral formula, we introduce some special exhaustion functions. Let (M, g) be a Riemannian manifold and let Φ be a Lipschitz continuous function on M^m satisfying the following conditions (see [9]):

(i) $\Phi \geqq 0$ and Φ is an exhaustion function of M, i.e., each sublevel set $B_\Phi(t) := \{\Phi < t\}$ is relatively compact in M for $t \geqq 0$.

(ii) $\Psi = \Phi^2$ is of class C^∞ and Ψ has only discrete critical points.

(iii) The constant $k_1 = \sup\limits_{x \in M} |\nabla \Phi|^2$ is finite.

The function Φ with properties (i), (ii) and (iii) will be called a special exhaustion function in the following sections.

3 The Results Under Exhaustion Functions

In this section, by using stress energy tensor, we derive some integral identities satisfied by differential forms $u \in A^p(M)$ which are solutions of $\triangle u - \lambda u = 0$, then we obtain a nonexistence theorem of p-eigenforms on manifolds with exhaustion functions which satisfy some pinching conditions.

Lemma 3.1 Let M be a complete Riemannian manifold with a special exhaustion function Φ. Assume $u \in A^p(M)$ satisfies $\triangle^p u = \lambda u$ and $\delta u = 0$. Let also $k \in R$ and $X \in \Gamma(TM)$ be a given vector field, then we have

$$\begin{aligned} & \int_{B_\Phi(R)} [\langle S_{du}, \nabla\theta_X\rangle - \frac{k}{4}|du|^2] dv_R - \lambda \int_{B_\Phi(R)} [\langle S_u, \nabla\theta_X\rangle - \frac{k}{4}|u|^2] dv_R \\ = & \int_{\partial B_\Phi(R)} [\frac{1}{2}(|du|^2 - \lambda|u|^2)\langle X, \nu\rangle - \langle i_X du + \frac{k}{4}u, i_\nu du\rangle + \lambda\langle i_X u, i_\nu u\rangle] d\sigma_R, \end{aligned}$$

where $d\sigma_R$ denotes the surface measure induced by dv_R on $\partial B_\Phi(R)$, ν denotes the outward unit normal to $\partial B_\Phi(R)$.

Proof By (2.3) and (2.6), we have

$$\int_{\partial B_\Phi(R)} S_\omega(X, \nu) d\sigma_R = \int_{B_\Phi(R)} [\langle S_\omega, \nabla\theta_X\rangle + \langle \delta\omega, i_X\omega\rangle + \langle i_X d\omega, \omega\rangle] dv_R.$$

Applying this relation first to $\omega = du$ and then to $\omega = u$, we obtain

$$\begin{aligned} & \int_{B_\Phi(R)} [\langle S_{du}, \nabla\theta_X\rangle - \lambda\langle S_u, \nabla\theta_X\rangle + \langle \delta du, i_X du\rangle - \lambda\langle i_X du, u\rangle] dv_R \\ = & \int_{\partial B_\Phi(R)} [S_{du}(X, \nu) - \lambda S_u(X, \nu)] d\sigma_R. \end{aligned}$$

By formula (2.1), taking into account that $\delta du = \triangle u = \lambda u$, we obtain

$$\begin{aligned} & \int_{B_\Phi(R)} [\langle S_{du}, \nabla\theta_X\rangle - \lambda\langle S_u, \nabla\theta_X\rangle] dv_R \\ = & \int_{\partial B_\Phi(R)} [S_{du}(X, \nu) - \lambda S_u(X, \nu)] d\sigma_R \\ = & \int_{\partial B_\Phi(R)} [\frac{1}{2}(|du|^2 - \lambda|u|^2)\langle X, \nu\rangle - \langle i_X du, i_\nu du\rangle + \lambda\langle i_X u, i_\nu u\rangle] d\sigma_R, \end{aligned}$$

and we have by Stokes's theorem

$$\begin{aligned} \int_{B_\Phi(R)} [|du|^2 - \lambda|u|^2] dv_R &= \int_{B_\Phi(R)} \langle u, \delta du - \lambda u\rangle dv_R + \int_{\partial B_\Phi(R)} \langle u, i_\nu du\rangle d\sigma_R \\ &= \int_{\partial B_\Phi(R)} \langle u, i_\nu du\rangle d\sigma_R. \end{aligned}$$

Thus

$$\begin{aligned} & \int_{B_\Phi(R)} [\langle S_{du}, \nabla\theta_X\rangle - \frac{k}{4}|du|^2] dv_R - \lambda \int_{B_\Phi(R)} [\langle S_u, \nabla\theta_X\rangle - \frac{k}{4}|u|^2] dv_R \\ = & \int_{\partial B_\Phi(R)} [\frac{1}{2}(|du|^2 - \lambda|u|^2)\langle X, \nu\rangle - \langle i_X du + \frac{k}{4}u, i_\nu du\rangle + \lambda\langle i_X u, i_\nu u\rangle] d\sigma_R. \end{aligned}$$

This proves Lemma 3.1.

Let $\{e_i\}$ be a local orthonormal frame field, and denote by L_X the Lie differentiation in the direction of X. We now specialize the discussion to the case that the metric g is the conformal deformation of the background metric g_0, as specified in the introduction, and obtain the following lemma.

Lemma 3.2 Assume that u satisfies the hypotheses of Lemma 3.1. Suppose further that $g = fg_0$ and M has a special exhaustion function Φ satisfying

$$\text{(i)}\ \liminf_{t\to+\infty}|\nabla_{g_0}\Psi|^2_{g_0}(t)[\max_{x\in\partial B_\Phi(t)}|f(x)|] \geq C > 0 \quad\text{and}\quad \text{(ii)}\ \int_1^{+\infty}\frac{1}{|\nabla_{g_0}\Psi|_{g_0}}dr = +\infty. \tag{3.1}$$

Denote by $X = \frac{\nabla_{g_0}\Psi}{2} = \Phi\nabla_{g_0}\Phi$, where ∇_{g_0} is the connection of g_0. Then, there exists a sequence $R_n \to +\infty$ such that, denoting by $B_\Phi(R)$ the exhaustion ball of radius R centered at a given x_0,

$$\begin{aligned}&\lim_{n\to\infty}\int_{B_\Phi(R_n)}\{\frac{1}{2}|du|^2(\mathrm{tr}L_Xg-k)-\sum_{s,t}\langle i_{e_s}du, i_{e_t}du\rangle_g(L_Xg)(e_s,e_t)\\&-\lambda[\frac{1}{2}|u|^2(trL_Xg-k)-\sum_{s,t}\langle i_{e_s}u, i_{e_t}u\rangle_g(L_Xg)(e_s,e_t)]\}dv_g = 0.\end{aligned} \tag{3.2}$$

Proof By (2.4) and the definition of Lie differentiation, we have

$$\begin{aligned}(L_Xg)(e_s,e_t) &= g(\nabla_{e_s}X, e_t) + g(e_s, \nabla_{e_t}X)\\&= \frac{1}{2}(\nabla\theta_{\nabla_{g_0}\Psi})(e_t,e_s) + \frac{1}{2}(\nabla\theta_{\nabla_{g_0}\Psi})(e_s,e_t),\end{aligned}$$

where ∇ is the connection of g, thus

$$\begin{aligned}&\frac{1}{2}|du|^2(\mathrm{tr}L_Xg-k)-\sum_{s,t}\langle i_{e_s}du, i_{e_t}du\rangle_g(L_Xg)(e_s,e_t)\\=\ &\langle\frac{1}{2}|du|^2g, \nabla\theta_{\nabla_{g_0}\Psi}\rangle_g - \langle du\odot du, \nabla\theta_{\nabla_{g_0}\Psi}\rangle_g - \frac{k}{2}|du|^2\\=\ &2[\langle S_{du}, \nabla\theta_X\rangle - \frac{k}{4}|du|^2].\end{aligned}$$

Considering that $(\nabla_{g_0}\Psi)|_{\partial B_\Phi(t)}$ is an outward normal vector field along $\partial B_\Phi(t)$ for a regular value $t > 0$, $(\nabla_{g_0}\Psi)_x = |\nabla_{g_0}\Psi|(x)\nu_1$ for each point $x \in \partial B_\Phi(t)$, where ν_1 denote the g_0-unit outward normal vector field of $\partial B_\Phi(t)$. Let ν_2 be the g-unit outward normal vector field of $\partial B_\Phi(t)$, then the following identities are easily verified

$$\nu_2 = f^{-1/2}\nu_1, \qquad |\nabla\Psi|^2_g = f^{-1}|\nabla_{g_0}\Psi|^2_{g_0}.$$

Thus

$$\begin{aligned}g(\nu_2, X) &= \frac{1}{2}f^{1/2}|\nabla_{g_0}\Psi|_{g_0},\\g(i_Xdu, i_{\nu_2}du) &= \frac{1}{2}f^{1/2}|\nabla_{g_0}\Psi|_{g_0}g(i_{\nu_2}du, i_{\nu_2}du),\end{aligned}$$

and

$$g(i_X u, i_{\nu_2} u) = \frac{1}{2} f^{1/2} |\nabla_{g_0} \Psi|_{g_0} g(i_{\nu_2} u, i_{\nu_2} u).$$

Denoting by $S(R)$ the boundary term of Lemma 3.1. Using the Cauchy-Schwarz inequality and assumption (3.1)(i), we estimate

$$\begin{aligned} |S(R)| &\leq |\nabla_{g_0}\Psi|_{g_0} \int_{\partial B_\Phi(R)} f^{1/2}\{\frac{3}{4}(|du|^2 + \lambda |u|^2) + \frac{k}{4f^{1/2}|\nabla_{g_0}\Psi|_{g_0}}|u||du|\} d\sigma_{g,R} \\ &\leq C'|\nabla_{g_0}\Psi|_{g_0} \int_{\partial B_\Phi(R)} f^{1/2}(|du|^2 + |u|^2) d\sigma_{g,R}, \end{aligned}$$

where C' depends only on C, $|\lambda|$ and k. Since $u \in \mathrm{dom}(\triangle^p)$, $|u|^2$ and $|du|^2$ are integrable on M (see [1, 2]). By the co-area formula, we have

$$\begin{aligned} & \int_0^{+\infty} dR \int_{\partial B_\Phi(R)} f^{1/2}(|du|^2 + |u|^2) d\sigma_{g,R} \\ = & \int_0^{+\infty} |\nabla_{g_0}\Phi|_{g_0} dR \int_{\partial B_\Phi(R)} \frac{1}{|\nabla_g \Phi|_g}(|du|^2 + |u|^2) d\sigma_{g,R} \\ \leq & \sqrt{k_1} \int_M (|du|^2 + |u|^2) dv_g < +\infty. \end{aligned}$$

By (3.1)(ii), we conclude that

$$\liminf_{R\to+\infty} S(R) = 0$$

as required. This proves Lemma 3.2.

Lemma 3.3 Maintaining the notation of Lemma 3.2. Denote by $A(x)$ (resp. $B(x)$) the smallest (resp. largest) eigenvalue of $\mathrm{Hess}_{g_0}(\Psi)$, that is, the Hessian of Ψ

$$A(x) g_0 \leq \mathrm{Hess}_{g_0}(\Psi) \leq B(x) g_0$$

holds on M in the sense of quadratic forms. Then for every p-form $u \in A^p(M) (p \geq 1)$, and every $k \in R$, we have

$$\begin{aligned} & \frac{|u|^2}{4}\{(m-2p) f^{-1}(\nabla_{g_0}\Psi)(f) - 2[pB - (m-p)A + k]\} \\ \leq & \frac{1}{2}|u|^2(\mathrm{tr} L_X g - k) - \sum_{s,t} \langle i_{e_s} u, i_{e_t} u\rangle_g (L_X g)(e_s, e_t) \\ \leq & \frac{|u|^2}{4}\{(m-2p) f^{-1}(\nabla_{g_0}\Psi)(f) - 2[pA - (m-p)B + k]\}, \end{aligned}$$

where $X = \frac{\nabla_{g_0}\Psi}{2} = \Phi \nabla_{g_0}\Phi$. If u is a 0-form, then we also have

$$\frac{1}{2}|u|^2(\mathrm{tr} L_X g - k) \geq \frac{|u|^2}{4}\{m f^{-1}(\nabla_{g_0}\Psi)(f) - 2[k - B - (m-1)A]\}.$$

Proof The proof is a modification of that of Lemma 1.3 in [3] (see also [10]), and we outline it here for completeness.

Since L_Xg is symmetric, the local orthonormal frame $\{e_s\}$ may be chosen in such a way that diagonalizes L_Xg. Let μ_s be the corresponding eigenvalues of L_Xg, so that $(L_Xg)(e_s,e_t)=\delta_{s,t}\mu_s$. We further assume that the indexing be chosen in such a way that $\mu_1\geq\mu_2\geq\cdots\geq\mu_m$. By definition of inner product, we may write

$$\begin{aligned}\sum_{s,t}\langle i_{e_s}u,i_{e_t}u\rangle_g(L_Xg)(e_s,e_t) &= \sum_s\langle i_{e_s}u,i_{e_s}u\rangle_g\mu_s\\ &= \frac{1}{(p-1)!}\sum_s\sum_{i_1,\cdots,i_{p-1}}|u(e_s,e_{i_1},\cdots,e_{i_{p-1}})|^2\mu_s\\ &= \frac{1}{p!}\sum_{i_1,\cdots,i_p}|u(e_{i_1},\cdots,e_{i_p})|^2\sum_{j=1}^p\mu_{i_j}.\end{aligned}$$

Since the eigenvalues are arranged in decreasing order, we have

$$\sum_{j=m-p+1}^m\mu_j\leq\sum_{j=1}^p\mu_{i_j}\leq\sum_{j=1}^p\mu_j$$

and we conclude that

$$|u|^2\sum_{j=m-p+1}^m\mu_j\leq\sum_{s,t}\langle i_{e_s}u,i_{e_t}u\rangle_gL_Xg(e_s,e_t)\leq|u|^2\sum_{j=1}^p\mu_j.$$

Denote by

$$Q=\frac{1}{2}|u|^2(\mathrm{tr}L_Xg-k)-\sum_{s,t}\langle i_{e_s}u,i_{e_t}u\rangle_g(L_Xg)(e_s,e_t).$$

Then we have

$$\frac{|u|^2}{2}(\sum_{i=p+1}^m\mu_i-\sum_{i=1}^p\mu_i-k)\leq Q\leq\frac{|u|^2}{2}(\sum_{i=1}^{m-p}\mu_i-\sum_{i=m-p+1}^m\mu_i-k). \tag{3.3}$$

By definition of Lie differentiation, we have

$$L_Xg=\frac{1}{2}(\nabla_{g_0}\Psi)(f)g_0+f\mathrm{Hess}_{g_0}(\Psi),$$

thus

$$[f^{-1}(\nabla_{g_0}\Psi)(f)+2A]g\leq L_{\nabla_{g_0}\Psi}g\leq[f^{-1}(\nabla_{g_0}\Psi)(f)+2B]g.$$

Therefore

$$\frac{1}{2}[f^{-1}(\nabla_{g_0}\Psi)(f)+2A]\leq\mu_i=\frac{1}{2}(L_{\nabla_{g_0}\Psi}g)(e_i,e_i)\leq\frac{1}{2}[f^{-1}(\nabla_{g_0}\Psi)(f)+2B].$$

The required conclusion now follows, substituting these estimates into (3.3). This proves Lemma 3.3.

Lemma 3.4 Maintaining the notation and assumption of Lemma 3.2. Assume that the functions $A(x)$, $B(x)$ satisfy $A(x) \geq \frac{m-2}{m}B(x)$ if $p=0$ and $A(x) \geq \frac{m-1}{m+1}B(x)$ if $p \geq 1$. Suppose also that

$$\begin{aligned}
&|f^{-1}(\nabla_{g_0}\Psi)(f)| \leq \frac{m}{m-1}[A-\frac{m-2}{m}B], && \text{if } p=0,\\
&|f^{-1}(\nabla_{g_0}\Psi)(f)| \leq \frac{m+1}{m-2p-1}[A-\frac{m-1}{m+1}B], && \text{if } 2 \leq 2p < m-2,\\
&f^{-1}(\nabla_{g_0}\Psi)(f) \geq -\frac{2(m+1)}{3}[A-\frac{m-1}{m+1}B], && \text{if } 2p=m-2 \text{ or } 2p=m,\\
&f^{-1}(\nabla_{g_0}\Psi)(f) \geq -(m+1)[A-\frac{m-1}{m+1}B], && \text{if } 2p=m-1,
\end{aligned}$$

and the above strict inequalities hold at some point $x_0 \in M$. If $u \in L^2(A^p(M))$ is such that $\delta u = 0$ and $\triangle^p u = \lambda u$ $(\lambda > 0)$, then $u = 0$.

Proof We consider the case $p \geq 1$. If $p = 0$, the argument is similar. By Lemma 3.3, we have

$$\begin{aligned}
&\frac{1}{2}|du|^2(\mathrm{tr}L_Xg-k)-\sum_{s,t}\langle i_{e_s}du, i_{e_t}du\rangle_g(L_Xg)(e_s,e_t)\\
\leq\ &\frac{|du|^2}{4}\{(m-2p-2)f^{-1}(\nabla_{g_0}\Psi)(f)-2[(p+1)A-(m-p-1)B+k]\}
\end{aligned} \tag{3.4}$$

and

$$\begin{aligned}
&\frac{1}{2}|u|^2(\mathrm{tr}L_Xg-k)-\sum_{s,t}\langle i_{e_s}u, i_{e_t}u\rangle_g(L_{\nabla_{g_0}\Psi}g)(e_s,e_t)\\
\geq\ &\frac{|u|^2}{4}\{(m-2p)f^{-1}(\nabla_{g_0}\Psi)(f)-2[pB-(m-p)A+k]\}.
\end{aligned} \tag{3.5}$$

Assume first that $2 \leq 2p < m-2$. We determine the constant k in such a way that

$$\frac{2}{m-2p-2}[(p+1)A-(m-p-1)B+k] = -\frac{2}{m-2p}[pB-(m-p)A+k].$$

Then a computation shows that the left hand side is equal to

$$\frac{m+1}{m-2p-1}[A-\frac{m-1}{m+1}B],$$

which is nonnegative by our assumption on A and B. Keeping into account the condition satisfied by f, we deduce that the right hand side of (3.4) is nonpositive, and that of (3.5) is nonnegative.

Arguing in a similar way, it is easily verified that the same conclusions hold if $2p$ is equal to $m-2$, $m-1$ or to m, provided we choose $k=-\frac{m-2}{6}A+\frac{m+2}{6}B$, $k=0$, $k=\frac{m-2}{6}A-\frac{m+2}{6}B$, respectively.

In all cases, the integrand in the left hand side of (3.2) is of constant (nonpositive) sign, and the integrals over the ball $B_\Phi(R_n)$ tend to the integral over M as n tends to $+\infty$. We conclude that the left hand side of (3.5) vanishes identically, and all inequalities are in fact equalities. In particular, when $2 \le 2p < m-2$,

$$\frac{|u|^2}{4}\{(m-2p)f^{-1}(\nabla_{g_0}\Psi)(f) - 2[pB-(m-p)A+k]\} = 0 \qquad \text{on } M,$$

thus

$$\frac{m-2p}{4}|u|^2\{f^{-1}(\nabla_{g_0}\Psi)(f) + \frac{m+1}{m-2p-1}[A-\frac{m-1}{m+1}B]\} = 0.$$

Now, by the continuity of f and the condition of the lemma, note that the quantity in braces on the left hand side is strictly positive in a neighbourhood of x_0. It follows that u must vanish in a neighbourhood of x_0. By unique continuation (see the proof of Lemma 1.4 of [3]), u must vanish identically on M, as required to finish the proof.

Theorem 3.1 Maintaining the notation and assumption of Lemma 3.3. Assume that the functions $A(x)$, $B(x)$ satisfy $A(x) \ge \frac{m-2}{m}B(x)$ if $p=0$ and $A(x) \ge \frac{m-1}{m+1}B(x)$ if $p \ge 1$. Suppose also that

$$\begin{aligned}
|f^{-1}(\nabla_{g_0}\Psi)(f)| &\le \frac{m}{m-1}[A-\frac{m-2}{m}B], && \text{if } p=0,\\
|f^{-1}(\nabla_{g_0}\Psi)(f)| &\le \frac{m+1}{m-2p+1}[A-\frac{m-1}{m+1}B], && \text{if } 2 \le 2p < m,\\
f^{-1}(\nabla_{g_0}\Psi)(f) &\ge -\frac{2(m+1)}{3}[A-\frac{m-1}{m+1}B], && \text{if } 2p = m,
\end{aligned}$$

and the above strict inequalities hold at some point $x_0 \in M$. If $u \in L^2(A^p(M))$ satisfies $\triangle^p u = \lambda u$ ($\lambda > 0$), then $u = 0$.

Proof The case $p=0$ can be deduced directly from Lemma 3.4. Thus, assume that $p \ge 1$, and let $u \in L^2(A^p(M))$ be such that $\triangle^p u = \lambda u$ with $\lambda > 0$. Then $v = \delta u$ belongs to $L^2(A^{p-1}(M))$ and satisfies $\triangle^{p-1} v = \lambda v$, $\delta v = 0$. It is readily verified that f satisfies the condition in Lemma 3.4 relative to $p-1$, so that $v = \delta u = 0$. But f also satisfies the condition of Lemma 3.4 relative to p, and therefore $u = 0$, as required.

Remark 3.1 When $f = 1$, so that there is no conformal deformation of the metric, the conditions of the theorem become to $A - \frac{m-2}{m}B > 0$ if $p = 0$ and $A - \frac{m-1}{m+1}B > 0$ if $p \ge 1$.

Remark 3.2 As in the case of harmonic p-forms, the conclusion for $p > m/2$ follows by Hodge duality.

4 The Results on Concrete Models

When we choose the special exhaustion functions to be the square of the intrinsic distance function, we have to consider the eigenvalue with respect to the radial direction, thus the discussion may be more complicate and finer.

Lemma 4.1 Let (M, g_0) be a complete Riemannian manifold with a pole o and let r be the distance function relative to o. Denote by $X = r\nabla_{g_0} r = r\partial r$. Assume that the radial sectional curvature of M satisfies $-\frac{a}{1+r^2} \leq K_r \leq \frac{b}{1+r^2}$ with $a \geq 0$, $b \in [0, 1/4]$. Let $g = fg_0$ be a conformally related metric. Then for every $u \in A^p(M)$ $(p \geq 1)$, and for every $k \in R$, we have

$$\begin{aligned} & r\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} + pB_2 - (m-p)B_1 - \min(B_2 - 1, 1 - B_1]\}|u|^2 \\ \leq\ & \frac{1}{2}|u|^2(\mathrm{tr}L_X g - k) - \sum_{s,t}\langle i_{e_s}u, i_{e_t}u\rangle_g (L_X g)(e_s, e_t) \\ \leq\ & r\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} - (m-p)B_2 + pB_1 + \min(B_2 - 1, 1 - B_1]\}|u|^2, \end{aligned}$$

where $B_1 = \frac{1+\sqrt{1-4b}}{2}$, $B_2 = \frac{1+\sqrt{1+4a}}{2}$.

If u is a 0-form, then we also have

$$\frac{1}{2}|u|^2(\mathrm{tr}L_X g - k) \geq r\{\frac{m}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} - (m-1)B_1 - 1]\}|u|^2.$$

Proof We consider the case $p \geq 1$. The statement relative to the case $p = 0$ can be proved in a similar way.

For $X = r\nabla_{g_0} r = r\partial r$, we have

$$L_X g_0 = \mathrm{Hess}_{g_0}(r^2) = 2r\mathrm{Hess}_{g_0} r + 2dr \otimes dr.$$

By $-\frac{a}{1+r^2} \leq K_r \leq \frac{b}{1+r^2}$ and [8], we have

$$\frac{\phi'}{\phi}[g_0 - dr \otimes dr] \leq \mathrm{Hess}_{g_0} r \leq \frac{\psi'}{\psi}[g_0 - dr \otimes dr].$$

Thus

$$2r\{\frac{\phi'}{\phi}g_0 + [\frac{1}{r} - \frac{\phi'}{\phi}]dr \otimes dr\} \leq L_X g_0 \leq 2r\{\frac{\psi'}{\psi}g_0 + [\frac{1}{r} - \frac{\psi'}{\psi}]dr \otimes dr\},$$

where ϕ, ψ are the solutions of the following problems, respectively

$$\begin{cases} \psi'' - \frac{a}{1+r^2}\psi = 0 \qquad \text{on } [0, +\infty), \\ \psi(0) = 0,\ \psi'(0) = 1, \end{cases}$$

and

$$\begin{cases} \phi'' + \frac{b}{1+r^2}\phi = 0 \qquad \text{on } [0, +\infty), \\ \phi(0) = 0,\ \phi'(0) = 1. \end{cases}$$

Standard comparison arguments show that

$$\frac{1+\sqrt{1-4b}}{2r} \leq \frac{\phi'}{\phi} \leq \frac{1}{r} \leq \frac{\psi'}{\psi} \leq \frac{1+\sqrt{1+4a}}{2r}.$$

Since L_X is a derivation, $L_X g = (Xf)g_0 + fL_X g_0$, thus

$$L_X g \geq r\{(f^{-1}\frac{\partial f}{\partial r} + 2\frac{\phi'}{\phi})g + 2(\frac{1}{r} - \frac{\phi'}{\phi})f dr \otimes dr\} \tag{4.1}$$

and

$$L_X g \le r\{(f^{-1}\frac{\partial f}{\partial r} + 2\frac{\psi'}{\psi})g + 2(\frac{1}{r} - \frac{\psi'}{\psi})f dr \otimes dr\}. \tag{4.2}$$

Choosing a local orthonormal frame $\{e_s\}$ which diagonalizes $L_X g$ and the corresponding eigenvalues satisfying $\mu_1 \ge \mu_2 \ge \cdots \ge \mu_m$. Further, if Y is g-orthogonal to ∂r, then $(L_X g)(Y, \partial r) = 0$, and we may therefore arrange that one of the vectors, say g_{s_r} be proportional to ∂r, that is $e_{s_r} = f^{-1/2}\partial r$. By (4.1) and (4.2), we obtain

$$\mu_s = r[f^{-1}\frac{\partial f}{\partial r} + \frac{2}{r}], \qquad \text{if} \quad s = s_r,$$

while

$$r[f^{-1}\frac{\partial f}{\partial r} + 2\frac{\phi'}{\phi}] \le \mu_s \le r[f^{-1}\frac{\partial f}{\partial r} + 2\frac{\psi'}{\psi}], \qquad \text{otherwise.}$$

By a discussion similar to Lemma 3.3, we have

$$\begin{aligned} &\frac{1}{2}|u|^2(\mathrm{tr}L_X g - k) - \sum_{s,t}\langle i_{e_s}u, i_{e_t}u\rangle_g (L_X g)(e_s, e_t) \\ \ge\ & r\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} + pr\frac{\psi'}{\psi} - (m-p)r\frac{\phi'}{\phi} - (r\frac{\psi'}{\psi} - 1)]\}|u|^2 \\ \ge\ & r\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} + pB_2 - (m-p)B_1 - (B_2 - 1)]\}|u|^2, \end{aligned} \tag{4.3}$$

if $s_r \le p$, and

$$\begin{aligned} &\frac{1}{2}|u|^2(\mathrm{tr}L_X g - k) - \sum_{s,t}\langle i_{e_s}u, i_{e_t}u\rangle_g (L_X g)(e_s, e_t) \\ \ge\ & r\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} + pr\frac{\psi'}{\psi} - (m-p)r\frac{\phi'}{\phi} - (1 - r\frac{\phi'}{\phi})]\}|u|^2 \\ \ge\ & r\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} + pB_2 - (m-p)B_1 - (1 - B_1)]\}|u|^2, \end{aligned} \tag{4.4}$$

if $s_r > p$. Thus

$$\begin{aligned} &\frac{1}{2}|u|^2(\mathrm{tr}L_X g - k) - \sum_{s,t}\langle i_{e_s}u, i_{e_t}u\rangle_g (L_X g)(e_s, e_t) \\ \ge\ & r\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} + pB_2 - (m-p)B_1 - \min(B_2 - 1, 1 - B_1]\}|u|^2. \end{aligned}$$

By the same discussion as above, we can also obtain

$$\begin{aligned} &\frac{1}{2}|u|^2(\mathrm{tr}L_X g - k) - \sum_{s,t}\langle i_{e_s}u, i_{e_t}u\rangle_g (L_X g)(e_s, e_t) \\ \le\ & r\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} - (m-p)B_2 + pB_1 + \min(B_2 - 1, 1 - B_1]\}|u|^2. \end{aligned}$$

This proves Lemma 4.1.

Lemma 4.2 Let (M, g_0) be a complete Riemannian manifold with a pole o and let r be the distance function relative to o. Assume that the radial sectional curvature of M

satisfies $-\frac{a}{1+r^2} \le K_r \le \frac{b}{1+r^2}$ with $a \ge 0$, $b \in [0, 1/4]$ and $\sqrt{1+4a}+\sqrt{1-4b} \ge 2$, $\sqrt{1-4b}-\sqrt{1+4a}+\frac{4}{m-1} \ge 0$ or $\sqrt{1+4a}+\sqrt{1-4b} \le 2$, $\sqrt{1-4b}-\frac{m-3}{m+1}\sqrt{1+4a} \ge 0$ if $p \ge 1$ and $\sqrt{1+4a}+\sqrt{1-4b} \le 2$, $\sqrt{1-4b}-\frac{m-2}{m}\sqrt{1+4a}+\frac{2}{m} \ge 0$ if $p=0$. Let $g = fg_0$ be a conformally related metric. Suppose also that

$$\begin{aligned}
&|f^{-1}\frac{\partial f}{\partial r}| \le \frac{1}{2}[\sqrt{1-4b}-\sqrt{1+4a}+\frac{4}{m-1}]r^{-1}, && \text{if } p=0,\\
&|f^{-1}\frac{\partial f}{\partial r}| \le \frac{m-1}{2(m-2p-1)}[\sqrt{1-4b}-\sqrt{1+4a}+\frac{4}{m-1}]r^{-1}, && \text{if } 2 \le 2p < m-2,\\
&f^{-1}\frac{\partial f}{\partial r} \ge -\frac{m-1}{4}[\sqrt{1-4b}-\sqrt{1+4a}+\frac{4}{m-1}]r^{-1}, && \text{if } 2p=m-2 \text{ or } 2p=m,\\
&f^{-1}\frac{\partial f}{\partial r} \ge -\frac{m-1}{2}[\sqrt{1-4b}-\sqrt{1+4a}+\frac{4}{m-1}]r^{-1}, && \text{if } 2p=m-1,
\end{aligned}$$

when $\sqrt{1+4a}+\sqrt{1-4b} \ge 2$, or

$$\begin{aligned}
&|f^{-1}\frac{\partial f}{\partial r}| \le \frac{m}{2(m-1)}(\sqrt{1-4b}-\frac{m-2}{m}\sqrt{1+4a}+\frac{2}{m})r^{-1}, && \text{if } p=0,\\
&|f^{-1}\frac{\partial f}{\partial r}| \le \frac{m+1}{2(m-2p-1)}(\sqrt{1-4b}-\frac{(m-3)\sqrt{1+4a}}{m+1})r^{-1}, && \text{if } 2 \le 2p < m-2,\\
&f^{-1}\frac{\partial f}{\partial r} \ge -\frac{m+1}{4}(\sqrt{1-4b}-\frac{(m-3)\sqrt{1+4a}}{m+1})r^{-1}, && \text{if } 2p=m-2 \text{ or } 2p=m,\\
&f^{-1}\frac{\partial f}{\partial r} \ge -\frac{m+1}{2}(\sqrt{1-4b}-\frac{(m-3)\sqrt{1+4a}}{m+1})r^{-1}, && \text{if } 2p=m-1,
\end{aligned}$$

when $\sqrt{1+4a}+\sqrt{1-4b} \le 2$. If $u \in L^2(A^p(M))$ is such that $\delta u = 0$ and $\triangle^p u = \lambda u$ ($\lambda > 0$), then $u = 0$.

Proof If $\sqrt{1+4a}+\sqrt{1-4b} \ge 2$, that is, $B_2 - 1 \ge 1 - B_1$, then for $du \in A^{p+1}(M)$, by Lemma 4.1, we have

$$\begin{aligned}
&\frac{1}{2}|du|^2(\mathrm{tr}L_Xg-k)-\sum_{s,t}\langle i_{e_s}du, i_{e_t}du\rangle_g(L_Xg)(e_s,e_t)\\
\le\ & r\{\frac{m-2p-2}{2}f^{-1}\frac{\partial f}{\partial r}-r^{-1}[\frac{k}{2}-(m-p-1)B_2+pB_1+1]\}|du|^2 \qquad (4.5)
\end{aligned}$$

and

$$\begin{aligned}
&\frac{1}{2}|u|^2(\mathrm{tr}L_Xg-k)-\sum_{s,t}\langle i_{e_s}u, i_{e_t}u\rangle_g(L_Xg)(e_s,e_t)\\
\ge\ & r\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r}-r^{-1}[\frac{k}{2}+pB_2-(m-p-1)B_1-1]\}|u|^2. \qquad (4.6)
\end{aligned}$$

Assume first that $2 \le 2p < m-2$. We determine the constant k in such a way that

$$\frac{2}{m-2p-2}[\frac{k}{2}-(m-p-1)B_2+pB_1+1] = -\frac{2}{m-2p}[\frac{k}{2}+pB_2-(m-p-1)B_1-1].$$

Then a computation shows that the left hand side is equal to

$$\frac{1}{m-2p-1}[(m-1)B_1-(m-1)B_2+2] = \frac{m-1}{2(m-2p+1)}[\sqrt{1-4b}-\sqrt{1+4a}+\frac{4}{m-1}],$$

which is nonnegative by our assumption. Keeping into account the condition satisfies by f, we deduce that the right hand side of (4.5) is nonpositive, and that of (4.6) is nonnegative. Therefore the integral in the left hand side of (3.2) is nonnegative. We conclude that the left hand side of (4.6) vanishes identically, and all inequalities are in fact equalities. In particular, when $2 \le 2p < m-2$, by (4.3), we have

$$r\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} + pr\frac{\psi'}{\psi} - (m-p)r\frac{\phi'}{\phi} - (1 - r\frac{\phi'}{\phi})]\}|u|^2 \equiv 0 \text{ on } M.$$

Now, note that the quantity in braces on the left hand side is strictly positive in a neighbourhood of o. Indeed, we may rewrite it in the form

$$\{\frac{m-2p}{2}f^{-1}\frac{\partial f}{\partial r} - r^{-1}[\frac{k}{2} + pB_2 - (m-p-1)B_1 - 1)]\} + p(B_2 - r\frac{\psi'}{\psi}) + (m-p-1)(r\frac{\phi'}{\phi} - B_1).$$

If $B_1 < 1$ or $B_2 > 1$, then the claim follows from the fact that $r\frac{\phi'}{\phi} \to 1$ and $r\frac{\psi'}{\psi} \to 1$ as $r \to 0$. If $B_1 = B_2 = 1$, then $a = b = 0$ and $\phi = \psi = r$, so the last two term are identically zero. But then

$$-[\frac{k}{2} + pB_2 - (m-p-1)B_1 - 1)] = \frac{m-2p}{m-2p-1},$$

and since $f^{-1}\partial f/\partial r$ is bounded in a neighbourhood of o (f being smooth and positive on M), the first term is strictly positive near o. By unique continuation (see [3]), u must vanish identically on M.

The other cases can be proved by the same way. This proves Lemma 4.2.

By the same discussion as Theorem 3.1, we have

Theorem 4.1 Let (M, g_0) be a complete Riemannian manifold with a pole o and let r be the distance function relative to o. Assume that the radial sectional curvature of M satisfies $-\frac{a}{1+r^2} \le K_r \le \frac{b}{1+r^2}$ with $a \ge 0$, $b \in [0, 1/4]$ and $\sqrt{1+4a} + \sqrt{1-4b} \ge 2$, $\sqrt{1-4b} - \sqrt{1+4a} + \frac{4}{m-1} \ge 0$ or $\sqrt{1+4a} + \sqrt{1-4b} \le 2$, $\sqrt{1-4b} - \frac{m-3}{m+1}\sqrt{1+4a} \ge 0$ if $p \ge 1$ and $\sqrt{1+4a} + \sqrt{1-4b} \le 2$, $\sqrt{1-4b} - \frac{m-2}{m}\sqrt{1+4a} + \frac{2}{m} \ge 0$ if $p = 0$. Let $g = fg_0$ be a conformally related metric. Suppose also that

$$|f^{-1}\frac{\partial f}{\partial r}| \le \frac{1}{2}[\sqrt{1-4b} - \sqrt{1+4a} + \frac{4}{m-1}]r^{-1}, \qquad \text{if } p = 0,$$

$$|f^{-1}\frac{\partial f}{\partial r}| \le \frac{m-1}{2(m-2p+1)}[\sqrt{1-4b} - \sqrt{1+4a} + \frac{4}{m-1}]r^{-1}, \qquad \text{if } 2 \le 2p < m,$$

$$f^{-1}\frac{\partial f}{\partial r} \ge -\frac{m-1}{4}[\sqrt{1-4b} - \sqrt{1+4a} + \frac{4}{m-1}]r^{-1}, \qquad \text{if } 2p = m,$$

when $\sqrt{1+4a} + \sqrt{1-4b} \ge 2$, or

$$|f^{-1}\frac{\partial f}{\partial r}| \le \frac{m}{2(m-1)}(\sqrt{1-4b} - \frac{m-2}{m}\sqrt{1+4a} + \frac{2}{m})r^{-1}, \qquad \text{if } p = 0,$$

$$|f^{-1}\frac{\partial f}{\partial r}| \le \frac{m+1}{2(m-2p+1)}(\sqrt{1-4b} - \frac{(m-3)\sqrt{1+4a}}{m+1})r^{-1}, \qquad \text{if } 2 \le 2p < m,$$

$$f^{-1}\frac{\partial f}{\partial r} \ge -\frac{m+1}{4}(\sqrt{1-4b} - \frac{(m-3)\sqrt{1+4a}}{m+1})r^{-1}, \qquad \text{if } 2p = m,$$

when $\sqrt{1+4a}+\sqrt{1-4b}\leq 2$. If $u\in L^2(A^p(M))$ satisfies $\triangle^p u=\lambda u$ $(\lambda>0)$, then $u=0$.

Remark 4.1 Using the above method, we can also obtain nonexistence theorems for those manifolds whose radial sectional curvature satisfies $-\frac{A}{(1+r^2)^{1+\epsilon}}\leq K_r\leq \frac{B}{(1+r^2)^{1+\epsilon}}$ with $\epsilon>0$, $A\geq 0$, $0\leq B<2\epsilon$, or $-\alpha^2\leq K_r\leq -\beta^2$ with $\alpha\geq 0$, $\beta\geq 0$.

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完备流形上拉普拉斯算子的L^2特征形式

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摘要: 本文研究了完备非紧流行上拉普拉斯算子的L^2特征形式. 利用应力能量张量的方法, 得到在此类流形上拉普拉斯算子的L^2特征形式的一些不存在性定理。

关键词: 应力能量张量; 微分形式; Hodge 拉普拉斯算子

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