Vol. 37 (2017)
No. 2

数 学 杂 志
J. of Math. (PRC)

THE POINTED LAYER SOLUTION OF SINGULAR PERTURBATION FOR NONLINEAR EVOLUTION EQUATIONS WITH TWO PARAMETERS

OUYANG Cheng[1], CHEN Xian-feng[2], MO Jia-qi[3]

(*1.Faculty of Science, Huzhou University, Huzhou 313000, China*)

(*2.Department of Mathematics, Shanghai Jiaotong University, Shanghai 200240, China*)

(*3.Department of Mathematic, Anhui Normal University, Wuhu 241003, China*)

Abstract: In this paper, the nonlinear singular perturbation problem for the evolution equations is studied. The outer solution and corrective terms of the pointed, boundary and initial layers for the solution are constructed. By using the fixed point theorem, the uniformly validity of solution to the problem ia proved and the results of the study for the singular perturbation with two parameters is extended.

Keywords: pointed layer; singular perturbation; evolution equation

2010 MR Subject Classification: 35B25

Document code: A **Article ID:** 0255-7797(2017)02-0247-10

1 Introduction

The nonlinear singular perturbation evolution equations are an important target in the mathematical, engineering mathematics and physical etc. circles. Many approximate methods were improved. Recently, many scholars did a great of work, such as de Jager et al. [1], Barbu et al. [2], Hovhannisyan et al. [3], Graef et al. [4], Barbu et al. [5], Bonfoh et al. [6], Faye et al. [7], Samusenko [8], Liu [9] and so on. Using the singular perturbation and other's theorys and methods the authoes also studied a class of nonlinear singular perturbation problems [10–24]. In this paper, using the special and simple method, we consider a class of the evolution equation.

Now we studied the following singular perturbation evolution equations initial-boundary value problem with two parameters

$$\varepsilon^2\frac{\partial^2 u}{\partial t^2}-\mu^2 Lu=f(t,x,u),\quad (t,x)\in(0,T_0]\times\Omega, \tag{1.1}$$

$$u=g(t,x),\quad x\in\partial\Omega, \tag{1.2}$$

$$u|_{t=0}=h_1(x),\quad \varepsilon\frac{\partial u}{\partial t}|_{t=0}=h_2(x),\quad x\in\Omega, \tag{1.3}$$

* **Received date:** 2015-06-18 **Accepted date:** 2015-11-30

Foundation item: Supported by the National Natural Science Foundation of China (11371248) and the Natural Science Foundation of Zhejiang Province, China (LY13A010005).

Biography: Ouyang Cheng (1962–), female, born at Deqing, Zhejiang, master, professor, major in Applied Mathematics.

where

$$L=\sum_{i,j=1}^{n}\alpha_{ij}\frac{\partial^2}{\partial x_i\partial x_j}+\sum_{i=1}^{n}\beta_i\frac{\partial}{\partial x_i},$$
$$\sum_{i,j=1}^{n}\alpha_{ij}\xi_i\xi_j\geq\lambda\sum_{i=1}^{n}\xi_i^2,\quad\forall\xi_i\in\Re,\ \lambda>0,$$

ε and μ are small positive parameters, $x=(x_1,x_1,\cdots,x_n)\in\Omega,\Omega$ is a bounded region in $\Re^n$, $\partial\Omega$ denotes boundary of Ω for class $C^{1+\alpha}$, where $\alpha\in(0,1)$ is Hölder exponent, T_0 is a positive constant large enough, $f(t,x,u)$ is a disturbed term, L signifies a uniformly elliptic operator.

Hypotheses that

$[H_1]$ $\sigma=\varepsilon/\mu$ as $\mu\to 0$;

$[H_2]$ α_{ij},β_i with regard to x are Hölder continuous, g and h_i are sufficiently smooth functions in correspondence ranges;

$[H_3]$ f is a sufficiently smooth functions in correspondence ranges except $x_0\in\Omega$;

$[H_4]$ $f(t,x,u)\leq -c<0,(x\neq x_0)$, where $C>0$ is a constant and for $f(t,x,u)=0$, there exists a solution U_{00}, such as $\lim\limits_{x\to x_0}U(t,x)\neq U(t,x_0)$.

2 Construct Outer Solution

Now we construct the outer solution of problems (1.1)–(1.3).

The reduced problem for the original problem is

$$f(t,x,u)=0. \tag{2.1}$$

From hypotheses, there is a solution $U_{00}(t,x)$ $(x\neq x_0)$ to equation (2.1). And there is a $U_{00}(t,x)$ which satisfies $f(t,x_0,U_{00}(t.x_0))=0$.

Let the outer solution $U_{00}(t,x)$ to problems (1.1)–(1.3), and

$$U(t,x)\sim\sum_{i,j=0}^{\infty}U_{ij}(t,x)\varepsilon^i\mu^j. \tag{2.2}$$

Substituting eq. (2.2) into eq. (1.1), developing the nonlinear term f in ε, and μ, and equating coefficients of the same powers of $\varepsilon^i\mu^j$ $(i,j=0,1,\cdots,i+j\neq 0)$, respectively. We can obtain $U_{ij}(t,x)$, $i,j=0,1,\cdots,i+j\neq 0$. Substituting $U_{00}(t,x)$ and $U_{ij}(t,x)$, $i,j=0,1,\cdots,i+j\neq 0$ into eq.(2.2), we obtain the outer solution $U(t,x)$ to the original problem. But it does not continue at (t,x_0) and it may not satisfy the boundary and initial conditions (1.2)–(1.3), so that we need to construct the pointed layer, boundary layer and initial layer corrective functions.

3 Construct Pointed Layer Corrective Term

Set up a local coordinate system (ρ, ϕ) near $x_0 \in \Omega$. Define the coordinate of every point Q in the neighborhood of x_0 with the following way: the coordinate $\rho(\leq \rho_0)$ is the distance from the point Q to x_0, where ρ_0 is small enough. The $\phi = (\phi_1, \phi_2, \cdots, \phi_{n-1})$ is a nonsingular coordinate.

In the neighborhood of $x_0 : (0 \leq \rho \leq \rho_0) \in \Omega$,

$$L = a_{nn}\frac{\partial^2}{\partial\rho^2} + \sum_{i=1}^{n-1} a_{ni}\frac{\partial^2}{\partial\rho\partial\phi_i} + \sum_{i,j=1}^{n-1} a_{ij}\frac{\partial^2}{\partial\phi_i\partial\phi_j} + b_n\frac{\partial}{\partial\rho} + \sum_{i=1}^{n-1} b_i\frac{\partial}{\partial\phi_i}, \tag{3.1}$$

where

$$a_{nn} = \sum_{i,j=1}^{n} \alpha_{ij}\frac{\partial\rho}{\partial x_i}\frac{\partial\rho}{\partial x_j}, \quad a_{ni} = 2\sum_{j,k=1}^{n} \alpha_{jk}\frac{\partial\rho}{\partial x_j}\frac{\partial\phi_i}{\partial x_k}, \quad a_{ij} = \sum_{k,l=1}^{n} \alpha_{kl}\frac{\partial\phi_i}{\partial x_k}\frac{\partial\phi_j}{\partial x_l},$$
$$b_n = \sum_{i,j=1}^{n} \alpha_{ij}\frac{\partial^2\rho}{\partial x_i\partial x_j}, \quad b_i = \sum_{i,j=1}^{n} \alpha_{ij}\frac{\partial_i^2\phi_i}{\partial x_i\partial x_j}.$$

We lead into the variables of multiple scales[1] on $(0 \leq \rho \leq \rho_0) \subset \Omega$:

$$\widetilde{\sigma} = \frac{h(\rho, \phi)}{\mu}, \quad \widetilde{\rho} = \rho \quad \widetilde{\phi} = \phi,$$

where $h(\rho, \phi)$ is a function to be determined. For convenience, we still substitute ρ, ϕ for $\widetilde{\rho}, \widetilde{\phi}$ below respectively. From eq.(3.1), we have

$$L = \frac{1}{\mu^2}K_0 + \frac{1}{\mu}K_1 + K_2, \tag{3.2}$$

while

$$K_0 = a_{nn}h_\rho^2\frac{\partial^2}{\partial\rho^2}$$

and K_1, K_2 are determined operators and their constructions are omitted.

Let $h_\rho = \sqrt{1/a_{nn}}$ and the solution u of original problems (1.1)–(1.3) be

$$u = U(t, x) + V_1(t, \rho, \phi), \tag{3.3}$$

where V_1 is a pointed layer corrective term. And

$$V_1 \sim \sum_{i,j=0}^{\infty} v_{1ij}(t, \rho, \phi)\sigma^i\mu^j. \tag{3.4}$$

Substituting eqs.(3.1)–(3.4) into eq.(1.1), expanding nonlinear terms in σ and μ, and equating the coefficients of like powers of $\sigma^i\mu^j$, respectively, for $i, j = 0, 1, \cdots$, we obtain

$$K_{10}v_{100} = 0, \quad (\rho, \phi) \in (0 \leq \rho \leq \rho_0), \tag{3.5}$$
$$v_{100}|_{\rho=0} = -U_{00}(t, x_0), \tag{3.6}$$
$$K_{10}v_{1ij} = G_{ij}, \quad (\rho, \phi) \in (0 \leq \rho \leq \rho_0), \quad i, j = 0, 1, \cdots, \quad i + j \neq 0, \tag{3.7}$$
$$v_{1ij}|_{\rho=0} = -U_{ij}(t, x_0), \quad i, j = 0, 1, \cdots, \quad i + j \neq 0, \tag{3.8}$$

where G_{ij} $(i,j = 0,1,\cdots, \quad i+j \neq 0)$ are determined functions. From problems (3.5)–(3.6), we can have v_{100}, From v_{100} and eqs.(3.7)–(3.8), we can obtain solutions v_{1ij} $(i,j = 0,1,\cdots, \quad i+j \neq 0)$, successively.

From the hypotheses, it is easy to see that v_{1ij} $(i,j = 0,1,\cdots)$ possesses boundary layer behavior

$$v_{1ij} = O(\exp(-\delta_{ij}\frac{\rho}{\sigma})), \quad i = 0,1,\cdots, \tag{3.9}$$

where $\delta_{ij} > 0$ $(i,j = 0,1,\cdots)$ are constants.

Let $\overline{v}_{1ij} = \psi(\rho)v_{1ij}$, where $\psi(\rho)$ is a sufficiently smooth function in $0 \leq \rho \leq \rho_0$, and satisfies

$$\psi(\rho) = \begin{cases} 1, & 0 \leq \rho \leq (1/3)\rho_0, \\ 0, & \rho \geq (2/3)\rho_0. \end{cases}$$

For convenience, we still substitute v_{1ij} for $\overline{v}_{1ij}$ below. Then from eq. (3.4), we have the pointed layer corrective term V_1 near $(0 \leq \rho \leq \rho_0) \subset \Omega$.

4 Construct Boundary Layer Corrective Term

Now we set up a local coordinate system $(\overline{\rho}, \overline{\phi})$ in the neighborhood near $\partial\Omega : 0 \leq \overline{\rho} \leq \overline{\rho}_0$ as Ref. [9], where $\overline{\phi} = (\overline{\phi}_1, \overline{\phi}_2, \cdots, \overline{\phi}_{n-1})$. In the neighborhood of $\partial\Omega : 0 \leq \overline{\rho} \leq \overline{\rho}_0$,

$$L = \overline{a}_{nn}\frac{\partial^2}{\partial\overline{\rho}^2} + \sum_{i=1}^{n-1}\overline{a}_{ni}\frac{\partial^2}{\partial\overline{\rho}\partial\overline{\phi}_i} + \sum_{i,j=1}^{n-1}\overline{a}_{ij}\frac{\partial^2}{\partial\overline{\phi}_i\partial\overline{\phi}_j} + \overline{b}_n\frac{\partial}{\partial\overline{\rho}} + \sum_{i=1}^{n-1}\overline{b}_i\frac{\partial}{\partial\overline{\phi}_i}, \tag{4.1}$$

where

$$\overline{a}_{nn} = \sum_{i,j=1}^{n}\alpha_{ij}\frac{\partial\overline{\rho}}{\partial x_i}\frac{\partial\overline{\rho}}{\partial x_j}, \quad \overline{a}_{ni} = 2\sum_{j,k=1}^{n}\alpha_{jk}\frac{\partial\overline{\rho}}{\partial x_j}\frac{\partial\overline{\phi_i}}{\partial x_k}, \quad \overline{a}_{ij} = \sum_{k,l=1}^{n}\alpha_{kl}\frac{\partial\overline{\phi}_i}{\partial x_k}\frac{\partial\overline{\phi_j}}{\partial x_l},$$

$$\overline{b}_n = \sum_{i,j=1}^{n}\alpha_{ij}\frac{\partial^2\overline{\rho}}{\partial x_i\partial x_j}, \quad \overline{b}_i = \sum_{i,j=1}^{n}\alpha_{ij}\frac{\partial_i^2\overline{\phi}_i}{\partial x_i\partial x_j}.$$

We lead into the variables of multiple scales [1] on $(0 \leq \overline{\rho} \leq \overline{\rho}_0) \subset \Omega$:

$$\overline{\sigma} = \frac{\overline{h}(\overline{\rho}, \overline{\phi})}{\mu}, \quad \widetilde{\rho} = \overline{\rho} \quad \widetilde{\phi} = \overline{\phi},$$

where $\overline{h}(\overline{\rho}, \overline{\phi})$ is a function to be determined. For convenience, we still substitute $\overline{\rho}, \overline{\phi}$ for $\widetilde{\rho}, \widetilde{\phi}$ below respectively. From (4.1), we have

$$L = \frac{1}{\mu^2}\overline{K}_0 + \frac{1}{\mu}\overline{K}_1 + \overline{K}_2, \tag{4.2}$$

while

$$\overline{K}_0 = \overline{a}_{nn}\overline{h}_{\overline{\rho}}^2\frac{\partial^2}{\partial\overline{\sigma}^2}$$

and $\overline{K}_1, \overline{K}_2$ are determined operators and their constructions are omitted too.

Let $\overline{h}_{\overline{\rho}} = \sqrt{1/\overline{a}_{nn}}$ and the solution u of original problems (1.1)–(1.3) be

$$u = U + V_2, \tag{4.3}$$

where V_2 is a boundary layer corrective term. And

$$V_2 \sim \sum_{i,j=0}^{\infty} v_{2ij}(t, \overline{\rho}, \overline{\phi})\varepsilon^i \sigma^j. \tag{4.4}$$

Substituting eq.(4.4) into eqs.(1.1) and (1.2), expanding nonlinear terms in ε and σ, and equating the coefficients of like powers of $\varepsilon^i\sigma^j$ $(i, j = 0, 1, \cdots)$. And we obtain

$$\overline{K}_0 v_{200} = 0, \quad (\overline{\rho}, \overline{\phi}) \in (0 \le \overline{\rho} \le \overline{\rho}_0), \tag{4.5}$$

$$v_{200}|_{\overline{\rho}=0} = g(t, x) - U_{00}(t, x), \tag{4.6}$$

$$K_0 v_{2ij} = \overline{G}_{ij}, \quad (\overline{\rho}, \overline{\phi}) \in (0 \le \overline{\rho} \le \overline{\rho}_0), \quad i, j = 0, 1, \cdots, \quad i + j \ne 0, \tag{4.7}$$

$$v_{2ij}|_{\overline{\rho}=0} = -U_{ij}(t, x_0), \quad i, j = 0, 1, \cdots, \quad i + j \ne 0, \tag{4.8}$$

where $\overline{G}_{ij}$ $(i, j = 0, 1, \cdots, \ i + j \ne 0)$ are determined functions successively, their constructions are omitted too.

From problems (4.5)–(4.6), we can have v_{200}. And from eqs. (4.7), (4.8), we can obtain solutions v_{2ij} $(i = 0, 1, \cdots, \ i+j \ne 0)$ successively. Substituting into eq. (4.4), we obtain the boundary layer corrective function V_2 for the original boundary value problems (1.1)–(1.3).

From the hypotheses, it is easy to see that v_{2ij} $(i, j = 0, 1, \cdots)$ possesses boundary layer behavior

$$v_{2ij} = O(\exp(-\overline{\delta}_{ij}\frac{\overline{\rho}}{\sigma})), \quad i, j = 0, 1, \cdots, \tag{4.9}$$

where $\overline{\delta}_{ij} > 0$ $(i, j = 0, 1, \cdots)$ are constants.

Let $\overline{v}_{2ij} = \overline{\psi}(\overline{\rho})v_{2ij}$, where $\overline{\psi}(\overline{\rho})$ is a sufficiently smooth function in $0 \le \overline{\rho} \le \overline{\rho}_0$, and satisfies

$$\overline{\psi}(\overline{\rho}) = \begin{cases} 1, & 0 \le \overline{\rho} \le (1/3)\overline{\rho}_0, \\ 0, & \overline{\rho} \ge (2/3)\overline{\rho}_0. \end{cases}$$

For convenience, we still substitute v_{2ij} for $\overline{v}_{2ij}$ below. Then from eq. (4.4) we have the boundary layer corrective term V_2 near $(0 \le \overline{\rho} \le \overline{\rho}_0)$.

5 Construct Initial Layer Corrective Term

The solution u of original problems (1.1)–(1.3) be

$$u = U + V_1 + V_2 + W, \tag{5.1}$$

where W is an initial layer corrective term. Substituting eq. (5.1) into eqs. (1.1)–(1.3), we have

$$
\begin{aligned}
&\varepsilon^2 W_{tt}-\mu^2 LW=f(t,x,U+V_1+V_2+W)\\
&-f(t,x,U+V_1+V_2)-\mu^2 L(U+V_1+V_2), &(5.2)\\
&W|_{x\in\partial\Omega}=(g(t,x)-U(t,x)-V_2(t,x))_{x\in\partial\Omega}, &(5.3)\\
&W|_{t=0}=h_1(x)-U(0,x)-V_1(0,x)-V_2(0,x),\quad x\in\Omega, &(5.4)\\
&\varepsilon\frac{\partial W}{\partial t}|_{t=0}=h_2(x)-\varepsilon(\frac{\partial U(0,x)}{\partial t}-\frac{\partial V_1(0,x)}{\partial t}-\frac{\partial V_2(0,x)}{\partial t}),\quad x\in\Omega. &(5.5)
\end{aligned}
$$

We lead into a stretched variable [1, 2]: $\tau=t/\varepsilon$ and let

$$W\sim\sum_{i,j=0}^{\infty}w_{ij}(\tau,x)\varepsilon^i\mu^j. \tag{5.6}$$

Substituting eqs. (2.2), (3.4), (4.4) and (5.6) into eqs. (5.2)–(5.5), expanding nonlinear terms in ε and μ, and equating the coefficients of like powers of $\varepsilon^i\mu^j$, respectively, for $i,j=0,1,\cdots,$ we obtain

$$
\begin{aligned}
&(w_{00})_{\tau\tau}=f(0,x,U_{00}+v_{100}+v_{200}+w_{00})-f(0,x,U_{00}+v_{100}+v_{200}), &(5.7)\\
&w_{00}|_{x\in\partial\Omega}=0, &(5.8)\\
&w_{00}|_{\tau=0}=h_1(x)-U_{00}(0,x)-v_{100}(0,x)-v_{200}(0,x),\quad x\in\Omega, &(5.9)\\
&\frac{\partial w_{00}}{\partial\tau}|_{\tau=0}=0,\quad x\in\Omega. &(5.10)\\
&(w_{ij})_{\tau\tau}=\overline{G}_{ij},\quad i,j=0,1,\cdots,\ i+j\neq0, &(5.11)\\
&w_{ij}|_{x\in\partial\Omega}=h_2(x),\quad i,j=0,1,\cdots,\ i+j\neq0, &(5.12)\\
&w_{ij}|_{\tau=0}=-U_{ij}(0,x)-v_{1ij}(0,x)-v_{2ij}(0,x),\\
&x\in\Omega,\quad i,j=0,1,\cdots,\ i+j\neq0, &(5.13)\\
&\frac{\partial w_{ij}}{\partial\tau}|_{\tau=0}=-\frac{\partial U_{(i-1)j}(0,x)}{\partial t}-\frac{\partial v_{1(i-1)j}(0,x)}{\partial t}-\frac{\partial v_{2(i-1)j}(0,x)}{\partial t},\\
&x\in\Omega,\quad i,j=0,1,\cdots,\quad i+j\neq0, &(5.14)
\end{aligned}
$$

where $\overline{G}_{ij}$ $(i,j=0,1,\cdots,\ i+j\neq0)$ are determined functions. From problems (5.7)–(5.10), we can have w_{00}, From w_{00} and eqs. (5.11)–(5.14), we can obtain solutions $w_{ij}(i,j=0,1,\cdots,\ i+j\neq0)$ successively.

From the hypotheses, it is easy to see that w_{ij} $(i,j=0,1,\cdots)$ possesses initial layer behavior

$$w_{ij}=O(\exp(-\widetilde{\delta}_{ij}\frac{t}{\varepsilon})),\quad i=0,1,\cdots, \tag{5.15}$$

where $\widetilde{\delta}_{ij}>0$ $(i,j=0,1,\cdots)$ are constants.

Then from eq. (5.15) we have the initial corrective term W.

From eq. (5.1), thus we obtain the formal asymptotic expansion of solution u for the nonlinear singular perturbation evolution equations initial-boundary value problems (1.1)–

(1.3) with two parameters

$$\begin{aligned} u \sim & U_{00} + \sum_{i,j=0,1,i+j\neq 0}^{\infty} U_{ij}\varepsilon^i\mu^j + \sum_{i,j=0}^{\infty}(v_{1ij}(t,x)+v_{2ij}(t,x))\sigma^i\mu^j \\ & + \sum_{i,j=0}^{\infty} w_{ij}(\tau,x)\varepsilon^i\mu^j, \quad 0<\varepsilon,\mu,\sigma \ll 1. \end{aligned} \tag{5.16}$$

6 The Main Result

Now we prove that this expansion (5.16) is a uniformly valid in Ω and we have the following theorem

Theorem Under hypotheses $[H_1]-[H_4]$, then there exists a solution $u(t,x)$ of the nonlinear singular perturbation evolution equation initial-boundary value problems (1.1)–(1.3) with two parameters and holds the uniformly valid asymptotic expansion (5.16) for ε and μ in $(t,x)\in[0,T_0]\times\overline{\Omega}$.

Proof We now get the remainder term $R(t,x)$ of the initial-boundary value problems (1.1)–(1.3). Let

$$u(t,x)=\overline{u}(t,x)+R(t,x), \tag{6.1}$$

where

$$\begin{aligned} \overline{u} \sim & U_{00} + \sum_{i,j=0,i+j\neq 0}^{m} U_{ij}\varepsilon^i\mu^j + \sum_{i,j=0}^{m}(v_{1ij}(t,x)+v_{2ij}(t,x))\sigma^i\mu^j \\ & + \sum_{i,j=0}^{m} w_{ij}(\tau,x)\varepsilon^i\mu^j. \end{aligned}$$

Using eqs. (2.2), (3.9), (4.9), (5.15), (6.1), we obtain

$$\begin{aligned} F[R] &\equiv \varepsilon^2\frac{\partial^2 R}{\partial t^2}-\mu^2 LR - f(t,x,\overline{u}+R)+f(t,x,\overline{u}) \\ &= O(\lambda^{m+1}) \ \ x\in\Omega,\ \ \lambda=\max(\varepsilon,\mu,\sigma), \\ R &= O(\lambda^{m+1}) \ \ x\in\partial\Omega,\ \ \lambda=\max(\varepsilon,\mu,\sigma), \\ R|_{t=0} &= O(\lambda^{m+1}) \ \ x\in\Omega,\ \ \lambda=\max(\varepsilon,\mu,\sigma), \\ \varepsilon\frac{\partial R}{\partial t}|_{t=0} &= O(\lambda^{m+1}) \ \ x\in\Omega,\ \ \lambda=\max(\varepsilon,\mu,\sigma). \end{aligned}$$

The linearized differential operator $\overline{L}$ reads

$$\overline{L}[p]=\varepsilon^2\frac{\partial^2 p}{\partial t^2}-\mu^2 L[p],$$

and therefore

$$\Psi[p]\equiv f[p]-\overline{L}[p]=f(t,x,\overline{u})-f(t,x,(\overline{u}+p))+f_u(t,x,(\overline{u}+p))p.$$

For fixed ε, μ, the normed linear space N is chosen as

$$N = \{p | p \in C^2((0, T_0] \times \Omega),\ p|_{\partial\Omega} = g,\ p|_{t=0} = h_1,\ p_t|_{t=0} = h_{12}\}$$

with norm

$$\|p\| = \max_{t \in (0.T_0], x \in \Omega} |p|,$$

and the Banach space B as

$$B = \{q | q \in C((0, T_0] \times \Omega)\}$$

with norm

$$\|q\| = \max_{t \in (0.T_0], x \in \Omega} |q|.$$

From the hypotheses we may show that the condition

$$\|L^{-1}[g]\| \leq l^{-1}\|g\|, \quad \forall g \in B$$

of the fixed point theorem [1, 2] is fulfilled where l^{-1} is independent of ε and μ, i.e., L^{-1} is continuous. The Lipschitz condition of the fixed point theorem become

$$\begin{aligned} & \|\Psi[p_2] - \Psi[p_1]\| \\ < \ & C_1 \max_{t \in (0,T_0], x \in \Omega} \{(|p|_1 + |p|_2)|p_2 - p_1|\} + C_2 \max_{t \in (0,T_0], x \in \Omega} \{|p^2| \cdot |p_2 - p_1|\} \\ < \ & C_3 t \|p_2 - p_1\|, \end{aligned}$$

where C_1, C_2 and C_3 are constants independent of ε and μ, this inequality is valid for all p_1, p_2 in a ball $K_N(r)$ with $\|r\| \leq 1$. Finally, we obtain the result that the remainder term exists and moreover

$$\max_{t \in (0,T_0], x \in \Omega} |R(t, x)| = O(\lambda^{m+1}), \quad \lambda = \max(\varepsilon, \mu, \sigma).$$

From eq. (6.1), we have

$$\begin{aligned} \overline{u} \ \equiv \ & U_{00} + \sum_{i,j=0, i+j\neq 0}^{m} U_{ij}\varepsilon^i \mu^j + \sum_{i,j=0}^{m} (v_{1ij}(t, x) + v_{2ij}(t, x))\sigma^i \mu^j \\ & + \sum_{i,j=0}^{m} w_{ij}(\tau, x)\varepsilon^i \mu^j + O(\lambda^{m+1}), \quad 0 < \varepsilon, \mu, \sigma., \quad \lambda = \max(\varepsilon, \mu, \sigma). \end{aligned}$$

The proof of the theorem is completed.

References

[1] de Jager E M, Jiang Furu. The theory of singular perturbation[M]. Amsterdam: North-Holland Publishing Co., 1996.

[2] Barbu L, Morosanu G. Singularly perturbed boundary-value problems[M]. Basel: Birkhauserm Verlag AG, 2007.

[3] Hovhannisyan G, Vulanovic R. Stability inequalities for one-dimensional singular perturbation problems[J]. Nonl. Stud., 2008, 15 (4): 297–322.

[4] Graef J R, Kong L. Solutions of second order multi-point boundary value problems[J]. Math. Proc. Camb. Philos. Soc., 2008, 145 (2): 489–510.

[5] Barbu L, Cosma E. Elliptic regularizations for the nonlinear heat equation[J]. J. Math. Anal. Appl., 2009, 351(2): 392–399.

[6] Bonfoh A, Grassrlli M, Miranville A. Intertial manifolds for a singular perturbation of the viscous Cahn-Hilliiard-Gurtin equation[J]. Topol. Meth. Nonl. Anal., 2010, 35(1): 155–185.

[7] Faye L, Frenod E, Seck D. Singularly perturbed degenerated parabolic equations and application to seabed morphodynamics in tided environment[J]. Discrete Contin. Dyn. Syst., 2011, 29 (3): 1001–1030

[8] Samusenko P F. Asymptotic integration of degenerate singularly perturbed systems of parabolic partial differential equations[J]. J. Math. Sci., 2013, 189 (5), 834–847.

[9] Mo Jiaqi. Singular perturbation for a class of nonlinear reaction diffusion systems[J]. Sci. China, Ser. A, 1989, 32 (11): 1306–1315.

[10] Mo Jiaqi, Lin Wantao. Asymptotic solution of activator inhibitor systems for nonlinear reaction diffusion equations[J]. J. Sys. Sci. Compl., 2008, 20 (1): 119–128.

[11] Mo Jiaqi. A class of singulaely perturbed differential-difference reaction diffusion equations[J]. Adv. Math., 2009, 38 (2): 227–230.

[12] Mo Jiaqi. Approximate solution of homotopic mapping to solitary for generalized nonlinear KdV system[J]. Chin. Phys. Lett., 2009, 26 (1): 010204-1-010204-4.

[13] Mo Jiaqi. A variational iteration solving method for a class of generalized Boussinesq equations[J]. Chin Phys. Lett., 2009, 26 (6): 060202-1-060202-3.

[14] Mo Jiaqi. Homotopic mapping solving method for gain fluency of laser pulse amplifier[J]. Sci. China, Ser. G, 2009, 39 (5): 568–661.

[15] Mo Jiaqi, Lin Wantao. Asymptotic solution for a class of sea-air oscillator model for El-Nino southern oscillation[J]. Chin. Phys., 2008, 17 (2): 370–372.

[16] Mo Jiaqi. A class of homotopic solving method for ENSO mpdel[J]. Acta Math. Sci., 2009, 29 (1): 101–109.

[17] Mo Jiaqi, Chen Huaijun. The corner layer solution of Robin problem for semilinear equation[J]. Math. Appl., 2012, 25(1): 1–4.

[18] Mo Jiaq, Lin Yihua, Lin Wantao, Chen Lihua. Perturbed solving method for interdecadal sea-air oscillator model[J]. Chin. Geogr. Sci., 2012, 22 (1): 1, 42–47.

[19] Mo Jiaqi, Yao Jingsun. A class of singularly perturbed nonlinear reaction diffusion equations with two parameters[J]. J. Math., 2011, 31(2): 341–346 (in Chinese).

[20] Ouyang Cheng, Cheng Lihua, Mo Jiaqi. Solving a class of burning disturbed problem with shock layers[J]. Chin. Phys. B, 2012, 21(5): 050203.

[21] Ouyang Cheng, Lin Wantao, Cheng Rongjun, Mo Jiaqi. A class of asymptotic solution of El Nino sea-Air time delay oscillator[J]. Acta. Phys. Sinica, 2013, 62(6): 060201 (in Chinese).

[22] Ouyang Cheng, Shi Janfang, Lin Wantao, Mo Jiaqi. Perturbation method of travelling solution for (2+1) dimensional disturbed time delay breaking solitary wave equation[J]. Acta. Phys. Sinica, 2013, 62(17): 170202 (in Chinese).

[23] Ouyang Cheng, Yao Jingsun, Shi Janfang, Mo Jiaqi. The solitary wave solution for a class of dusty plasma[J]. Acta. Phys. Sinica, 2013, 63(11): 110203 (in Chinese).

[24] Chen Lihua, Yao Jingsun, Wen Zhaohui, Mo Jiaqi. Shock wave solution for the nonlinear reaction diffusion problem[J]. J. Math., 2013, 33(3): 381–387.

两参数非线性发展方程的奇摄动尖层解

欧阳成[1], 陈贤峰[2], 莫嘉琪[3]

(1.湖州师范学院理学院, 浙江 湖州 313000)

(2.上海交通大学数学系, 上海 200240)

(3.安徽师范大学数学系, 安徽 芜湖 241003)

摘 要: 本文研究了一类具有非线性发展方程奇摄动问题. 引入伸长变量和多重尺度, 构造了初始边值问题外部解和尖层、边界层和初始层校正项, 得到了问题形式解. 利用不动点定理, 证明了问题的解的一致有效性. 推广了对两参数的奇摄动问题的研究结果.

关键词: 尖层; 奇摄动; 发展方程

MR(2010)**主题分类号**: 35B25 **中图分类号**: O175.4