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GORENSTEIN FLAT (COTORSION) DIMENSIONS AND HOPF ACTIONS

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Abstract: In this paper, we study the relationship of Gorenstein flat (cotorsion) dimensions between A-Mod and $A\#H$-Mod. Using the properties of separable functors, we get that (1) Let A be a right coherent ring, assume that $A\#H/A$ is separable and $\varphi : A \to A\#H$ is a splitting monomorphism of (A, A)-bimodules, then $l.Gwd(A) = l.Gwd(A\#H)$; (2) Assume that $A\#H/A$ is separable and $\varphi : A \to A\#H$ is a splitting monomorphism of (A, A)-bimodules, then $l.Gcd(A) = l.Gcd(A\#H)$, which generalized the results in skew group rings.

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1 Introduction

The (Gorenstein) homological properties and representation dimensions for skew group algebras, or more generally, for smash products and crossed products were discussed by several authors, for example in [4, 13, 14, 16, 17, 20]. In [13], López-Ramos studied the relationship of Gorenstein injective (projective) dimensions between A-Mod and $A\#H$-Mod. He showed that under some conditions, $glGid(A) < \infty$ if and only if $glGid(A\#H) < \infty$ (resp. $glGpd(A) < \infty$ if and only if $glGpd(A\#H) < \infty$).

The aim of this paper is to study the relationship of Gorenstein flat (cotorsion) dimensions between A-Mod and $A\#H$-Mod. First we prove that over a right coherent ring A, if $A\#H/A$ is separable and $\varphi : A \to A\#H$ is a splitting monomorphism of (A, A)-bimodules, $l.Gwd(A) = l.Gwd(A\#H)$. Then we study the relationship of Gorenstein cotorsion dimensions between A-Mod and $A\#H$-Mod. We prove that if $A\#H/A$ is separable and $\varphi : A \to A\#H$ is a splitting monomorphism of (A, A)-bimodules, $l.Gcd(A) = l.Gcd(A\#H)$.

Next we recall some notions and facts required in the following.

Throughout this paper, H always denotes a finite-dimensional Hopf algebra over k with comultiplication $\Delta : H \otimes H \to H$, counit $\varepsilon : H \to k$ and antipode $S : H \to H$. A k-algebra A is called a left H-module algebra if A is a left H-module such that $h \cdot (ab) = \sum (h_{(1)} \cdot a)(h_{(2)} \cdot b)$ and $h \cdot 1_A = \varepsilon(h)1_A$ for all $a, b \in A$ and $h \in H$.

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Let A be a left H-module algebra, the smash product algebra (or semidirect product) of A with H, denoted by $A\#H$, is the vector space $A\otimes H$, whose elements are denoted by $a\#h$ instead of $a\otimes h$, with multiplication given by $(a\#h)(b\#l)=\Sigma a(h_{(1)}\cdot b)\#h_{(2)}l$ for $a,b\in A$ and $h,l\in H$. The unit of $A\#H$ is $1\#1$ and we usually view ah as $a\#h$ and ha as $(1\#h)(a\#1)$. In this paper, A-Mod and $A\#H$-Mod denote the categories of left A-modules and left $A\#H$-modules, respectively.

The notion of separable functor was introduced in [15]. Consider categories $\mathcal{C}$ and $\mathcal{D}$, a covariant functor $F:\mathcal{C}\to\mathcal{D}$ is said to be separable if for all M,N in $\mathcal{C}$ there are maps $\varphi^F_{M,N}:\mathrm{Hom}_{\mathcal{D}}(F(M),F(N))\to\mathrm{Hom}_{\mathcal{C}}(M,N))$ satisfying the following conditions.

1. For $\alpha\in\mathrm{Hom}_{\mathcal{C}}(M,N)$, we have $\varphi^F_{M,N}(F(\alpha))=\alpha$.

2. Given $M',N'\in\mathcal{C}$, $\alpha\in\mathrm{Hom}_{\mathcal{C}}(M,M')$, $\beta\in\mathrm{Hom}_{\mathcal{C}}(N,N')$, $f\in\mathrm{Hom}_{\mathcal{D}}(F(M),F(N))$ and $g\in\mathrm{Hom}_{\mathcal{D}}(F(M'),F(N'))$ such that the following diagram commutes

$$\begin{tikzcd} F(M) \arrow[r, "f"] \arrow[d, "F(\alpha)"'] & F(N) \arrow[d, "F(\beta)"'] \\ F(M') \arrow[r, "g"] & F(N'), \end{tikzcd}$$

then the following diagram is also commutative

$$\begin{tikzcd} M \arrow[r, "\varphi^F_{M,N}(f)"] \arrow[d, "\alpha"'] & N \arrow[d, "\beta"'] \\ M' \arrow[r, "\varphi^F_{M',N'}(g)"] & N'. \end{tikzcd}$$

Let $\varphi:\ A\to A\#H$ denote the inclusion map. We can associate to φ the restriction of scalars functor ${}_A(-)$: $A\#H$-Mod $\to A$-Mod, the induction functor $A\#H\otimes_A - = Ind(-)$: A-Mod $\to A\#H$-Mod and the coinduction functor $\mathrm{Hom}_A(A\#H,-)$: A-Mod $\to A\#H$-Mod. It is well known that $A\#H\otimes_A -$ is left adjoint to ${}_A(-)$ and that $\mathrm{Hom}_A(A\#H,-)$ is right adjoint to ${}_A(-)$. Since H is a finite-dimensional Hopf algebra, by [6, Theorem 5], the functor $A\#H\otimes_A -$ is isomorphic to $\mathrm{Hom}_A(A\#H,\ -)$. So we have a double adjunctions $(A\#H\otimes_A -,\ {}_A(-))$ and $({}_A(-),\ A\#H\otimes_A -)$. Now we consider the separability of functors ${}_A(-)$ and $A\#H\otimes_A -$. From [15, Proposition 1.3], we have the following

1. ${}_A(-)$ is separable if and only if $A\#H/A$ is separable.

2. $A\#H\otimes_A - = \mathrm{Ind}(-)$ is separable if and only if φ splits as an A-bimodule map.

A left R-module M is called Gorenstein flat [7] if there exists an exact sequence

$$\cdots\longrightarrow F^{-2}\longrightarrow F^{-1}\longrightarrow F^0\longrightarrow F^1\longrightarrow\cdots$$

of flat left R-modules such that $M=\ker(F^0\to F^1)$ and which remains exact whenever $E\otimes_R -$ is applied for any injective right R-module E. We will say that M has Gorenstein flat dimension less than or equal to n [10] if there exists an exact sequence

$$0\longrightarrow F_n\longrightarrow F_{n-1}\longrightarrow\cdots\longrightarrow F_0\longrightarrow M\longrightarrow 0$$

with every F_i being Gorenstein flat. If no such finite sequence exsits, define $Gfd_R(M) = \infty$; otherwise, if n is the least such integer, define $Gfd_R(M) = n$. In [3] left weak Gorenstein global dimension of R was define as $l.Gwd(R)$=sup$\{Gfd_R(M) \mid M$ is any left R-module$\}$. A left R-module M is called Gorenstein cotorsion [8] if $Ext^1_R(N, M) = 0$ for all Gorenstein flat left R-modules N. We will say that M has Gorenstein cotorsion dimension less than or equal to n [12] if there exists an exact sequence

$$0 \longrightarrow M \longrightarrow C^0 \longrightarrow C^1 \longrightarrow \cdots \longrightarrow C^n \longrightarrow 0$$

with every C^i being Gorenstein cotorsion. The left global Gorenstein cotorsion dimension $l.Gcd(R)$ of R is defined as the supremum of the Gorenstein cotorsion dimensions of left R-modules.

2 Gorenstein Flat Modules and Actions of Finite-Dimensional Hopf Algebras

In this paper, $\varphi:\ A \to A\#H$ always denotes the inclusion map. If $M \in A\#H$-Mod, then ${}_AM$ will denote the image of M by the restriction of the scalars functor ${}_A(-) : A\#H$-Mod$\to A$-Mod.

Lemmma 2.1 (see [11, Corollary 3.6A]) Let $\eta:\ R \to S$ be a ring homomorphism such that S becomes a flat left R-module under η. Then, for any injective module M_S, the right R-module M (obtained by pullback along η) is also injective.

Remark 2.2 Let $\varphi:\ A \to A\#H$ be the inclusion map. Since $A\#H$ is free as a left A-module, then from Lemma 2.1 we know that for any injective right $A\#H$-module M, the right A-module M (obtained by pullback along φ) is also injective.

Proposition 2.3 (1) If $M \in A$-Mod is Gorenstein flat, then $A\#H \otimes_A M$ is Gorenstein flat as a left $A\#H$-module.

(2) If $M \in A\#H$-Mod is Gorenstein flat, then ${}_AM$ is Gorenstein flat as a left A-module.

Proof

(1) Since M is a Gorenstein flat left A-module, we have an exact sequence

$$\mathfrak{F} \equiv\ \cdots \longrightarrow F^{-2} \longrightarrow F^{-1} \longrightarrow F^0 \longrightarrow F^1 \longrightarrow \cdots$$

of flat left A-modules such that $M = \ker(F^0 \to F^1)$ and which remains exact whenever $E \otimes_A -$ is applied for any injective right A-module E.

Since $A\#H$ is free as a right A-module by [5, Proposition 6.1.7] and $A\#H \otimes_A -$ preserves flat modules, we get that $A\#H \otimes_A \mathfrak{F}$ is an exact sequence of flat left $A\#H$-modules and

$$A\#H \otimes_A M = \ker(A\#H \otimes_A F^0 \to A\#H \otimes_A F^1).$$

Finally, let E' be any injective right $A\#H$-module. Then $E' \otimes_{A\#H} (A\#H \otimes_A \mathfrak{F}) \cong (E' \otimes_{A\#H} A\#H) \otimes_A \mathfrak{F}$ is exact since $E' \otimes_{A\#H} A\#H \cong E'$ (as right A-modules) is injective by Remark 2.2. Thus $A\#H \otimes_A M$ is Gorenstein flat.

(2) Let $M \in A\#H$-Mod be Gorenstein flat, then we have an exact sequence

$$\mathfrak{F}' \equiv \ \cdots \longrightarrow F'^{-2} \longrightarrow F'^{-1} \longrightarrow F'^{0} \longrightarrow F'^{1} \longrightarrow \cdots$$

of flat left $A\#H$-modules such that $M = \ker(F'^0 \to F'^1)$ and which remains exact whenever $E \otimes_{A\#H} -$ is applied for any injective right $A\#H$-module E. Then ${}_A\mathfrak{F}'$ is an exact sequence of flat left A-modules since the functor ${}_A(-)$ is exact and preserves flat modules.

Finally, let E' be any injective right A-module. Then

$$E' \otimes_A ({}_A\mathfrak{F}') \cong E' \otimes_A (A\#H \otimes_{A\#H} \mathfrak{F}') \cong (E' \otimes_A A\#H) \otimes_{A\#H} \mathfrak{F}' \ (*).$$

Since H is a finite-dimensional Hopf algebra, by [6, Theorem 5], we can easily get that $E' \otimes_A A\#H$ is injective as a right $A\#H$-module. By $(*)$ we know that $E' \otimes_A ({}_A\mathfrak{F}')$ is exact. Therefore ${}_AM$ is Gorenstein flat.

Proposition 2.4 Assume that $A\#H/A$ is separable and $\varphi : A \to A\#H$ is a splitting monomorphism of (A, A)-bimodules. Then A is a right coherent ring if and only if $A\#H$ is a right coherent ring.

Proof Let $\{F_i\}_{i\in I}$ be a family of flat left $A\#H$-modules, then ${}_A(F_i)$ is flat as a left A-module for every i. If we consider the adjoint pair $(A\#H \otimes_A -,\ {}_A(-))$, we know that ${}_A(-)$ preserves inverse limits. Thus ${}_A(\prod F_i) \cong \prod_A(F_i)$. Since A is a right coherent ring, ${}_A(\prod F_i) \cong \prod_A(F_i)$ is flat as a left A-module. Then, we get that $\prod F_i$ is a flat left $A\#H$-module. Thus $A\#H$ is a right coherent ring.

Conversely, let $\{F_i\}_{i\in I}$ be a family of flat left A-modules, since $A\#H \otimes_A -$ preserves flat modules, we know that $A\#H \otimes_A F_i$ is flat as a left $A\#H$-module for every i. If we consider the adjoint pair $({}_A(-),\ A\#H \otimes_A -)$, we know that $A\#H \otimes_A -$ preserves inverse limits. Thus

$$A\#H \otimes_A (\prod F_i) \cong \prod A\#H \otimes_A F_i.$$

Since $A\#H$ is a right coherent ring, $A\#H \otimes_A (\prod F_i) \cong \prod A\#H \otimes_A F_i$ is flat as a left $A\#H$-module. Then, we get that ${}_A(A\#H \otimes_A \prod F_i)$ is a flat left A-module. Since $\varphi : A \to A\#H$ is a splitting monomorphism of (A, A)-bimodules, we get that the functor $A\#H \otimes_A -$ is separable by [15, Proposition 1.3]. Consider the adjoint pair $(\ A\#H \otimes_A -,\ {}_A(-))$, by [9, Proposition 5] we know that the natural map $\eta_M : M \to_A (A\#H \otimes_A M)$ is a split monomorphism for every $M \in A$-Mod. Then $\prod F_i$ is a direct summand of ${}_A(A\#H \otimes_A \prod F_i)$. Hence $\prod F_i$ is flat as a left A-module since the class of flat modules is closed under direct summands. Thus A is a right coherent ring.

Next we consider the relationship of the left weak Gorenstein global dimensions in A-Mod and $A\#H$-Mod when A is right coherent.

Theorem 2.5 Let A be a right coherent ring. Assume that $A\#H/A$ is separable and $\varphi : A \to A\#H$ is a splitting monomorphism of (A, A)-bimodules. Then $l.Gwd(A) = l.Gwd(A\#H)$.

Proof For every n, we need to show that $Gfd_A(M) \le n$ for every left A-module M if and only if $Gfd_{A\#H}(N) \le n$ for every left $A\#H$-module N.

Suppose that $l.Gwd(A\#H) = n$ and let M be any A-module. From Proposition 2.3 we know that $A\#H \otimes_A -$ and ${}_A(-)$ both preserve Gorenstein flat modules. Thus

$$Gfd_{A\#H}(A\#H \otimes_A M) \leq n, \;\; Gfd_A({}_A(A\#H \otimes_A M)) \leq n.$$

Since $A\#H \otimes_A -$ is separable, M is a direct summand of ${}_A(A\#H \otimes_A M)$. Since A is a right coherent ring, by [2, Propositions 2.2 and 2.10] we know that $Gfd_A(M) \leq n$.

Since $A\#H/A$ is separable, ${}_A(-)$ is separable by [15, Proposition 1.3]. Similarly, we can prove that if $l.Gwd(A) \leq n$ then $l.Gwd(A\#H) \leq n$.

Lemma 2.6 (1) If $N \in A$-Mod is Gorenstein cotorsion, then $A\#H \otimes_A N$ is Gorenstein cotorsion as a left $A\#H$-Mod.

(2) If $N \in A\#H$-Mod is Gorenstein cotorsion, then ${}_AN$ is Gorenstein cotorsion as a left A-module.

(3) Let $M \in A\#H$-Mod and $A\#H/A$ be separable. Then M is Gorenstein cotorsion as a left $A\#H$-module if and only if ${}_AM$ is Gorenstein cotorsion as a left A-module.

Proof (1) Let N be any Gorenstein cotorsion left A-module and F any Gorenstein flat left $A\#H$-module. For F we have an exact sequence $0 \longrightarrow K \longrightarrow P \longrightarrow F \longrightarrow 0$ $(*)$ of left $A\#H$-modules with P projective. Since ${}_A(-)$ is exact and preserves Gorenstein flat and projective modules, we have an exact sequence $0 \longrightarrow {}_AK \longrightarrow {}_AP \longrightarrow {}_AF \longrightarrow 0$ with ${}_AP$ projective and ${}_AF$ Gorenstein flat. Hence we have the following commutative diagram:

$$\begin{tikzcd}
0 \arrow[r] & \mathrm{Hom}_{A\#H}(F, A\#H \otimes_A N) \arrow[r] \arrow[d, "\sigma_1"'] & \mathrm{Hom}_{A\#H}(P, A\#H \otimes_A N) \arrow[r] \arrow[d, "\sigma_2"'] & \mathrm{Hom}_{A\#H}(K, A\#H \otimes_A N) \arrow[d, "\sigma_3"'] & \\
0 \arrow[r] & \mathrm{Hom}_A({}_AF, N) \arrow[r] & \mathrm{Hom}_A({}_AP, N) \arrow[r] & \mathrm{Hom}_A({}_AK, N) \arrow[r] & 0.
\end{tikzcd}$$

Note that σ_1, σ_2 and σ_3 are isomorphisms by adjoint isomorphism. Hence

$$\mathrm{Hom}_{A\#H}(P, A\#H \otimes_A N) \to \mathrm{Hom}_{A\#H}(K, A\#H \otimes_A N)$$

is an epimorphism.

Applying the functor $\mathrm{Hom}_{A\#H}(-, A\#H \otimes_A N)$ to $(*)$, we get a long exact sequence

$$0 \to \mathrm{Hom}_{A\#H}(F, A\#H \otimes_A N) \longrightarrow \mathrm{Hom}_{A\#H}(P, A\#H \otimes_A N) \longrightarrow \mathrm{Hom}_{A\#H}(K, A\#H \otimes_A N)$$

$$\longrightarrow Ext^1_{A\#H}(F, A\#H \otimes_A N) \to Ext^1_{A\#H}(P, A\#H \otimes_A N) = 0.$$

Since

$$\mathrm{Hom}_{A\#H}(P, A\#H \otimes_A N) \to \mathrm{Hom}_{A\#H}(K, A\#H \otimes_A N)$$

is an epimorphism, we know that $Ext^1_{A\#H}(F, A\#H \otimes_A N) = 0$ for any Gorenstein flat left $A\#H$-module F. Hence $A\#H \otimes_A N$ is a Gorenstein cotorsion left $A\#H$-module.

(2) Similarly, using the adjoint pair $(A\#H \otimes_A -, \ {}_A(-))$ we can prove that ${}_A(-)$ preserves Gorenstein cotorsion modules.

(3) The "only if" part can be gotten directly by (2).

Conversely, if${}_AM$ is Gorenstein cotorsion as a left A-module, then by (1) we know that $A\#H\otimes_A {}_AM$ is Gorenstein cotorsion. Since $A\#H/A$ is separable, the functor ${}_A(-)$ is separable by [15, Proposition 1.3]. Consider the adjoint pair $({}_A(-),\ A\#H\otimes_A -)$, by [9, Proposition 5] we know that the natural map $\eta_M : M \to A\#H\otimes_A {}_AM$ is a split monomorphism for every left $A\#H$-module M. Then M is a direct summand of $A\#H\otimes_A {}_AM$. Hence M is Gorenstein cotorsion as a left $A\#H$-module since the class of Gorenstein cotorsion modules is closed under direct summands.

Proposition 2.7 Let $M \in A\#H$-Mod and $N \in A$-Mod. Then

(1) $Gcd_A({}_AM) \le Gcd_{A\#H}(M)$.

(2) $Gcd_{A\#H}(A\#H\otimes_A N) \le Gcd_A(N)$.

Proof (1) Assume that $Gcd_{A\#H}(M) = n < \infty$, then there exists an exact sequence of left $A\#H$-modules

$$0 \longrightarrow M \longrightarrow C^0 \longrightarrow C^1 \longrightarrow \cdots \longrightarrow C^n \longrightarrow 0$$

with every C^i being Gorenstein cotorsion. By Lemma 2.6, ${}_A(-)$ preserves Gorenstein cotosion modules, we have an exact sequence of left A-modules

$$0 \longrightarrow {}_AM \longrightarrow {}_AC^0 \longrightarrow {}_AC^1 \longrightarrow \cdots \longrightarrow {}_AC^n \longrightarrow 0$$

with every C^i being Gorenstein cotorsion. Thus $Gcd_A({}_AM) \le Gcd_{A\#H}(M)$.

(2) Similarly, using Lemma 2.6, we can get that

$$Gcd_{A\#H}(A\#H\otimes_A N) \le Gcd_A(N).$$

Theorem 2.8 Assume that $A\#H/A$ is separable and $\varphi : A \to A\#H$ is a splitting monomorphism of (A, A)-bimodules, then $l.Gcd(A) = l.Gcd(A\#H)$.

Proof Let M be any left A-module. Since $\varphi : A \to A\#H$ is a splitting monomorphism of (A, A)-bimodules, M is a direct summand of ${}_A(A\#H\otimes_A M)$. Hence

$$Gcd_A(M) \le Gcd_A({}_A(A\#H\otimes_A M)).$$

By Proposition 2.7,

$$Gcd_A({}_A(A\#H\otimes_A M)) \le Gcd_{A\#H}(A\#H\otimes_A M) \le l.Gcd(A\#H).$$

Thus $l.Gcd(A) \le l.Gcd(A\#H)$.

Let N be any left $A\#H$-module. Since $A\#H/A$ is separable, N is a direct summand of $A\#H\otimes_A {}_AN$. Hence

$$Gcd_{A\#H}(N) \le Gcd_{A\#H}(A\#H\otimes_A {}_AN).$$

By Proposition 2.7,

$$Gcd_{A\#H}(A\#H\otimes_A {}_AN) \le Gcd_A({}_AN) \le l.Gcd(A).$$

Thus $l.Gcd(A\#H) \leq l.Gcd(A)$.

Corollary 2.9 Let A be a k-algebra and G a finite group with $|G|^{-1} \in k$. Then $l.Gcd(A) = l.Gcd(A * G)$.

Proof By the definition of the skew group ring, we know that A is a left H-module algebra and $A * G = A\#H$, where $H = kG$. Since G a finite group with $|G|^{-1} \in k$, H is semisimple. Then from [19], we know that $A\#H/A$ is separable. By [1, Lemma 4.5], we know that A is a direct summand of $A\#H$ as (A, A)-bimodule. By Theorem 2.8 we immediately get the desired result.

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Gorenstein平坦(余挠)维数和Hopf作用

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摘 要: 设H是域k上的有限维Hopf代数, A是左H-模代数. 本文研究了Gorenstein平坦(余挠)维数在A-模范畴和$A\#H$-模范畴之间的关系. 利用可分函子的性质, 证明了(1) 设A是右凝聚环, 若$A\#H/A$可分且$\varphi : A \to A\#H$是可裂的(A, A)-双模同态, 则 $l.Gwd(A) = l.Gwd(A\#H)$; (2) 若$A\#H/A$可分且$\varphi : A \to A\#H$是可裂的(A, A)-双模同态, 则 $l.Gcd(A) = l.Gcd(A\#H)$, 推广了斜群环上的结果.

关键词: 凝聚环; Gorenstein平坦模; Gorenstein余挠模

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