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MARCINKIEWICZ-ZYGMUND INEQUALITY FOR PAIRWISE NQD RANDOM VARIABLES AND ITS APPLICATIONS

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Abstract: In this paper, the author studies the Marcinkiewicz-Zygmund inequality for pairwise negative quadrant dependent (NQD) random variables and it applications. By using the truncated method, the author obtains the Marcinkiewicz-Zygmund inequality with exponent p ($1 \le p < 2$) for pairwise NQD random variables. As applications, the author obtains the simpler proofs of two L^r convergence results for pairwise NQD random variables, which improve the corresponding work by Chen [10] and Sung [20] respectively.

Keywords: pairwise NQD random variable; Marcinkiewicz-Zygmund inequality; L^r convergence

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1 Introduction

We first recall some concepts of dependent structure. The following famous concept of negative quadrant dependent (NQD) was introduced by Lehmann [1].

Definition 1.1 Two random variables X and Y are said to be NQD if

$$P(X \le x, Y \le y) \le P(X \le x)P(Y \le y) \quad \text{for all } x \text{ and } y.$$

A sequence of random variables $\{X_n, n \ge 1\}$ is said to be pairwise NQD if every pair of random variables in the sequence are NQD.

Definition 1.2 The random variables $X_1, \cdots, X_k$ are said to be negatively upper orthant dependent (NUOD) if for all real $x_1, \cdots, x_k$,

$$P(X_i > x_i, i = 1, 2, \cdots, k) \le \prod_{i=1}^{k} P(X_i > x_i),$$

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and negatively lower orthant dependent (NLOD) if

$$P(X_i \le x_i, i = 1, 2, \cdots, k) \le \prod_{i=1}^{k} P(X_i \le x_i).$$

Random variables $X_1, \cdots, X_k$ are said to be negatively orthant dependent (NOD) if they are both NUOD and NLOD. This concept was introduced by Ebrahimi and Ghosh [2].

Joag-Dev and Proschan [3] introduced the concept of negatively associated (NA).

Definition 1.3 A finite family of random variables $\{X_k, 1 \le k \le n\}$ is said to be NA if for any disjoint subsets A and B of $\{1, 2, \cdots, n\}$ and any real coordinatewise nondecreasing functions f on R^A and g on R^B,

$$\mathrm{Cov}(f(X_i, i \in A), g(X_j, j \in B)) \le 0,$$

whenever the covariance exists. An infinite family of random variables is NA if every finite subfamily is NA.

The concept of linearly negative quadrant dependent (LNQD) random variables was introduced by Newman [4].

Definition 1.4 A sequence $\{X_n, n \ge 1\}$ of random variables is said to be LNQD if for any disjoint subsets $A, B \subset \mathbf{Z}^+$ and positive $r_j^{'}s$, $\sum\limits_{k \in A} r_k X_k$ and $\sum\limits_{j \in B} r_j X_j$ are NQD.

Remark 1.1 It is important to note that NOD, NA or LNQD implies pairwise NQD.

It is well known that sequences of pairwise NQD random variables are a family of very wide scope and have been an attractive research topic in the recent papers. We refer reader to Matula [5], Wu [6], Liang et al. [7], Cabrera and Volodin [8], Wan [9], Chen [10], Li and Yang [11], Baek et al. [12], Gan and Chen [13], Meng and Lin [14], Baek and Park [15], Gan and Chen [16], Wu and Jiang [17], Wu and Guan [18], Wu and Wang [19], Sung [20].

As we know, moment inequalities are very important tools in establishing the limit theorems for sequences of random variables. For pairwise NQD random variables, the following Marcinkiewicz-Zygmund inequality with exponent 2

$$E\left|\sum_{k=1}^{n} X_k\right|^2 \le \sum_{k=1}^{n} E|X_k|^2 \tag{1.1}$$

was proved by Wu [6] (see Lemma 2.2). However, due to the limitation of the exponent 2 in inequality (1.1), many authors could not obtain desirable results of the convergence properties for pairwise NQD random variables.

According to our knowledge, the following Marcinkiewicz-Zygmund inequality with exponent p $(1 < p < 2)$

$$E\left|\sum_{k=1}^{n} X_k\right|^p \le C \sum_{k=1}^{n} E|X_k|^p \tag{1.2}$$

for pairwise NQD random variables has not been discussed in previous literature. It is not in doubt that inequality (1.2) has stronger application value. In this article, we will prove the above inequality (1.2) remains true for pairwise NQD random variables.

Recently Sung [20] obtained a L^r convergence result for weighted sums of arrays of rowwise pairwise NQD random variables.

Theorem A Let $\{X_{ni}, u_n \le i \le v_n, n \ge 1\}$ be an array of rowwise pairwise NQD random variables and $1 \le r < 2$. Let $\{a_{ni}, u_n \le i \le v_n, n \ge 1\}$ be an array of constants. Suppose that

(i) $\sup\limits_{n\ge1} \sum\limits_{i=u_n}^{v_n} |a_{ni}|^r E|X_{ni}|^r < \infty$,

(ii) $\sum\limits_{i=u_n}^{v_n} |a_{ni}|^r E|X_{ni}|^r I(|a_{ni}|^r |X_{ni}|^r > \varepsilon) \to 0$ as $n \to \infty$ for any $\varepsilon > 0$. Then

$$\sum_{i=u_n}^{v_n} a_{ni}(X_{ni} - EX_{ni}) \to 0$$

in L^r and, hence, in probability as $n \to \infty$.

Chen [10] presented the following L^r convergence result for sequence of pairwise NQD random variables.

Theorem B Let $\{X_n, n \ge 1\}$ be a sequence of pairwise NQD random variables with $EX_n = 0$ for all $n \ge 1$ and $1 \le r < 2$. $S_n = \sum\limits_{i=1}^{n} X_i$. Suppose that

$$\lim_{a\to\infty} \sup_{n\ge1} n^{-1} \sum_{i=1}^{n} E|X_i|^r I(|X_i| > a) = 0, \tag{1.3}$$

then $n^{-1/r} S_n \to 0$ in L^r as $n \to \infty$.

In this work, we first establish the Marcinkiewicz-Zygmund inequality with exponent p $(1 < p < 2)$ for pairwise NQD random variables. As applications, Theorem A and Theorem B are proved by some methods which are much simpler than those in Chen [10] and Sung [20].

Throughout this paper, the symbol C represents positive constants whose values may change from one place to another.

2 Main Result

To prove our main result, we need the following technical lemmas.

Lemma 2.1 (see [1]) Let $\{X_n, n \ge 1\}$ be a sequence of pairwise NQD random variables. Let $\{f_n, n \ge 1\}$ be a sequence of increasing functions. Then $\{f_n(X_n), n \ge 1\}$ is a sequence of pairwise NQD random variables.

Lemma 2.2 (see [6]) Let $\{X_n, n \ge 1\}$ be a sequence of pairwise NQD random variables with mean zero and $EX_n^2 < \infty$. Then $E|\sum\limits_{k=1}^{n} X_k|^2 \le \sum\limits_{k=1}^{n} E|X_k|^2$.

Now we present the Marcinkiewicz-Zygmund inequality with exponent p $(1 < p < 2)$ for pairwise NQD random variables.

Theorem 2.1 Let $\{X_n, n \ge 1\}$ be a sequence of pairwise NQD random variables with mean zero and $E|X_n|^p < \infty$ for $1 \le p < 2$. Then there exists a positive constant C

depending only on p, such that

$$E\Big|\sum_{k=1}^{n} X_k\Big|^p \le C\sum_{k=1}^{n} E|X_k|^p. \tag{2.1}$$

Proof Let $\varphi_n=\sum\limits_{k=1}^{n} E|X_k|^p$. For all $t\ge 1$, let

$$\begin{aligned}
Y_k &= -\varphi_n^{1/p}t^{1/p}I(X_k<-\varphi_n^{1/p}t^{1/p})+X_kI(|X_k|\le\varphi_n^{1/p}t^{1/p})+\varphi_n^{1/p}t^{1/p}I(X_k>\varphi_n^{1/p}t^{1/p}),\\
Z_k &= X_k-Y_k \;=\; (X_k+\varphi_n^{1/p}t^{1/p})I(X_k<-\varphi_n^{1/p}t^{1/p})+(X_k-\varphi_n^{1/p}t^{1/p})I(X_k>\varphi_n^{1/p}t^{1/p}).
\end{aligned}$$

By Lemma 2.1, it follows that $\{Y_k, k\ge 1\}$ and $\{Z_k, k\ge 1\}$ are sequences of pairwise NQD random variables. Then

$$\begin{aligned}
&E\Big|3^{-1}\varphi_n^{-1/p}\sum_{k=1}^{n} X_k\Big|^p\\
&=\int_0^\infty P\Big(\Big|\sum_{k=1}^{n} X_k\Big|\ge 3\,\varphi_n^{1/p}\,t^{1/p}\Big)\mathrm{d}\;\le\; 1+\int_1^\infty P\Big(\Big|\sum_{k=1}^{n} X_k\Big|\ge 3\,\varphi_n^{1/p}\,t^{1/p}\Big)\mathrm{d}t\\
&\le 1+\sum_{k=1}^{n}\int_1^\infty P\big(|X_k|>\varphi_n^{1/p}t^{1/p}\big)\mathrm{d}t+\int_1^\infty P\Big(\Big|\sum_{k=1}^{n} Y_k\Big|\ge 3\,\varphi_n^{1/p}\,t^{1/p}\Big)\mathrm{d}t\\
&=:1+I_1+I_2.
\end{aligned}$$

Noting that $\int_1^\infty P\big(|X_k|>\varphi_n^{1/p}t^{1/p}\big)\mathrm{d}t\le\varphi_n^{-1}E|X_k|^pI(|X_k|>\varphi_n^{1/p})$. Hence,

$$I_1\le\varphi_n^{-1}\sum_{k=1}^{n}E|X_k|^pI(|X_k|>\varphi_n^{1/p})\;\le\;1.$$

By $EX_k=0$ and $p\ge 1$, we have

$$\begin{aligned}
&\sup_{t\ge1}\varphi_n^{-1/p}t^{-1/p}\Big|\sum_{k=1}^{n}EY_k\Big| = \sup_{t\ge1}\varphi_n^{-1/p}t^{-1/p}\Big|\sum_{k=1}^{n}\big\{-\varphi_n^{1/p}t^{1/p}P(X_k<-\varphi_n^{1/p}t^{1/p})\\
&\quad+EX_kI(|X_k|\le\varphi_n^{1/p}t^{1/p})+\varphi_n^{1/p}t^{1/p}P(X_k>\varphi_n^{1/p}t^{1/p})\big\}\Big|\\
&=\sup_{t\ge1}\varphi_n^{-1/p}t^{-1/p}\Big|\sum_{k=1}^{n}\big\{-\varphi_n^{1/p}t^{1/p}P(X_k<-\varphi_n^{1/p}t^{1/p})\\
&\quad-EX_kI(|X_k|>\varphi_n^{1/p}t^{1/p})+\varphi_n^{1/p}t^{1/p}P(X_k>\varphi_n^{1/p}t^{1/p})\big\}\Big|\\
&\le\sup_{t\ge1}\varphi_n^{-1/p}t^{-1/p}\sum_{k=1}^{n}\big\{\varphi_n^{1/p}t^{1/p}P(|X_k|>\varphi_n^{1/p}t^{1/p})+E|X_k|I(|X_k|>\varphi_n^{1/p}t^{1/p})\big\}\\
&\quad\le\sup_{t\ge1}\sum_{k=1}^{n}P(|X_k|>\varphi_n^{1/p}t^{1/p})+\sup_{t\ge1}\varphi_n^{-1}t^{-1}\sum_{k=1}^{n}E|X_k|^pI(|X_k|>\varphi_n^{1/p}t^{1/p})\\
&\quad\le 2\varphi_n^{-1}\sum_{k=1}^{n}E|X_k|^pI(|X_k|>\varphi_n^{1/p})\;\le\;2.
\end{aligned}$$

Therefore, $\big|\sum\limits_{k=1}^{n} EY_k\big| \le 2\,\varphi_n^{1/p}t^{1/p}$ holds uniformly for $t \ge 1$. Then

$$I_2 \le \int_1^{\infty} P\Big(\Big|\sum_{k=1}^{n}(Y_k - EY_k)\Big| \ge \varphi_n^{1/p}t^{1/p}\Big)\mathrm{d}t.$$

By the Markov inequality, Lemma 2.2 and C_r-inequality, we have

$$\begin{aligned}
I_2 \le&\varphi_n^{-2/p}\int_1^{\infty} t^{-2/p}E\Big|\sum_{k=1}^{n}(Y_k - EY_k)\Big|^2\mathrm{d}t\\
\le&\varphi_n^{-2/p}\sum_{k=1}^{n}\int_1^{\infty} t^{-2/p}E(Y_k-EY_k)^2\mathrm{d}t \ \le\ 2\,\varphi_n^{-2/p}\sum_{k=1}^{n}\int_1^{\infty} t^{-2/p}EY_k^2\mathrm{d}t\\
=&2\,\varphi_n^{-2/p}\sum_{k=1}^{n}\int_1^{\infty} t^{-2/p}EX_k^2I(|X_k|\le\varphi_n^{1/p}t^{1/p})\mathrm{d}t + 2\sum_{k=1}^{n}\int_1^{\infty}P(|X_k|>\varphi_n^{1/p}t^{1/p})\mathrm{d}t\\
=&2\,\varphi_n^{-2/p}\sum_{k=1}^{n}\int_1^{\infty} t^{-2/p}EX_k^2I(|X_k|\le\varphi_n^{1/p})\mathrm{d}t + 2\sum_{k=1}^{n}\int_1^{\infty}P(|X_k|>\varphi_n^{1/p}t^{1/p})\mathrm{d}t\\
&+2\,\varphi_n^{-2/p}\sum_{k=1}^{n}\int_1^{\infty} t^{-2/p}EX_k^2I(\varphi_n^{1/p}<|X_k|\le\varphi_n^{1/p}t^{1/p})\mathrm{d}t\\
=&\ :I_3+I_4+I_5.
\end{aligned}$$

By a similar argument as in the proof of $I_1 \le 1$, we can prove $I_4 \le 2$. By $p<2$, we get

$$\begin{aligned}
I_3 =& \frac{2p}{2-p}\varphi_n^{-2/p}\sum_{k=1}^{n}EX_k^2I(|X_k|\le\varphi_n^{1/p})\\
\le& \frac{2p}{2-p}\varphi_n^{-1}\sum_{k=1}^{n}E|X_k|^pI(|X_k|\le\varphi_n^{1/p}) \ \le\ \frac{2p}{2-p}.
\end{aligned}$$

Finally we estimate I_5. Noting that $\sum\limits_{m=s}^{\infty} m^{-2/p} \le 2/(2-p)s^{1-2/p}$ and $(s+1)/s \le 2$ for all $s \ge 1$. We can get

$$\begin{aligned}
I_5 =& 2\,\varphi_n^{-2/p}\sum_{k=1}^{n}\sum_{m=1}^{\infty}\int_m^{m+1} t^{-2/p}EX_k^2I(\varphi_n^{1/p}<|X_k|\le\varphi_n^{1/p}t^{1/p})\mathrm{d}t\\
\le& 2\,\varphi_n^{-2/p}\sum_{k=1}^{n}\sum_{m=1}^{\infty} m^{-2/p}EX_k^2I(\varphi_n<|X_k|^p\le\varphi_n(m+1))\\
=& 2\,\varphi_n^{-2/p}\sum_{k=1}^{n}\sum_{m=1}^{\infty} m^{-2/p}\sum_{s=1}^{m}EX_k^2I(\varphi_n\,s<|X_k|^p\le\varphi_n(s+1))\\
=& 2\,\varphi_n^{-2/p}\sum_{k=1}^{n}\sum_{s=1}^{\infty}EX_k^2I(\varphi_n\,s<|X_k|^p\le\varphi_n(s+1))\sum_{m=s}^{\infty}m^{-2/p}\\
\le& \frac{4}{2-p}\,\varphi_n^{-2/p}\sum_{k=1}^{n}\sum_{s=1}^{\infty}s^{1-2/p}EX_k^2I(\varphi_n\,s<|X_k|^p\le\varphi_n(s+1))
\end{aligned}$$

$$\leq \frac{2^{2/p+1}}{2-p}\,\varphi_n^{-1}\sum_{k=1}^{n}\sum_{s=1}^{\infty}E|X_k|^pI(\varphi_n\,s<|X_k|^p\leq\varphi_n(s+1))$$
$$=\frac{2^{2/p+1}}{2-p}\,\varphi_n^{-1}\sum_{k=1}^{n}E|X_k|^pI(|X_k|^p>\varphi_n)\ \leq\ \frac{2^{2/p+1}}{2-p}.$$

From $I_1\leq 1$, $I_3\leq 2\,p/(2-p)$, $I_4\leq 2$ and $I_5\leq \frac{2^{2/p+1}}{2-p}$, we have

$$E\left|3^{-1}\varphi_n^{-1/p}\sum_{k=1}^{n}X_k\right|^p\ \leq\ 4+\frac{2\,p}{2-p}+\frac{2^{2/p+1}}{2-p}.$$

Let $C=3^p\big(4+\frac{2\,p}{2-p}+\frac{2^{2/p+1}}{2-p}\big)$. Clearly C depends only on p, then we get

$$E\left|\sum_{k=1}^{n}X_k\right|^p\ \leq\ C\sum_{k=1}^{n}E|X_k|^p.$$

The proof is completed.

Remark 2.1 Inequality (2.1) for NA and NOD random variables was proved by Shao [21] and Asadian et al. [22], respectively. The above result shows that the famous Marcinkiewicz-Zygmund inequality with exponent p $(1\leq p<2)$ remains true for pairwise NQD random variables.

3 Applications

As applications, we prove Theorem A and Theorem B by some simpler methods compared with Chen [10] and Sung [20].

Proof of Theorem A Without loss of generality, we may assume that $a_{ni}\geq 0$. For $u_n\leq i\leq v_n$, $n\geq 1$, let

$$Y_{ni}=-\varepsilon^{1/r}I(a_{ni}X_{ni}<-\varepsilon^{1/r})+a_{ni}X_{ni}I(a_{ni}|X_{ni}|\leq\varepsilon^{1/r})+\varepsilon^{1/r}I(a_{ni}X_{ni}>\varepsilon^{1/r}),$$
$$Z_{ni}=a_{ni}X_{ni}-Y_{ni}.$$

By Lemma 2.1, $\{Y_{ni},u_n\leq i\leq v_n,n\geq 1\}$ and $\{Z_{ni},u_n\leq i\leq v_n,n\geq 1\}$ are arrays of rowwise pairwise NQD. Given $\varepsilon>0$, by Theorem 2.1, we have

$$\begin{aligned}
&E\left|\sum_{i=u_n}^{v_n}a_{ni}(X_{ni}-EX_{ni})\right|^r\\
&\leq 2^{r-1}\left\{E\left|\sum_{i=u_n}^{v_n}(Z_{ni}-EZ_{ni})\right|^r+E\left|\sum_{i=u_n}^{v_n}(Y_{ni}-EY_{ni})\right|^r\right\}\\
&\leq 2^{r-1}E\left|\sum_{i=u_n}^{v_n}(Z_{ni}-EZ_{ni})\right|^r+2^{r-1}\left\{E\left|\sum_{i=u_n}^{v_n}(Y_{ni}-EY_{ni})\right|^2\right\}^{r/2}\\
&\leq C\,2^{r-1}\sum_{i=u_n}^{v_n}E|Z_{ni}|^r+C\,2^{r-1}\left\{\sum_{i=u_n}^{v_n}EY_{ni}^2\right\}^{r/2}\\
&=:I_6+I_7.
\end{aligned}$$

Noting that $|Z_{ni}| \le a_{ni}|X_{ni}|I(a_{ni}^r|X_{ni}|^r > \varepsilon)$. By condition (ii), we have

$$I_6 \ \le \ C \sum_{i=u_n}^{v_n} a_{ni}^r E|X_{ni}|^r I(a_{ni}^r|X_{ni}|^r > \varepsilon) \to 0 \quad \text{as} \ \ n \to \infty.$$

Next we prove $I_7 \to 0$ as $n \to \infty$. Without loss of generality, we may assume $0 < \varepsilon < 1$. Then

$$\begin{aligned} I_7^{2/r} &\le C \sum_{i=u_n}^{v_n} a_{ni}^2 E X_{ni}^2 I(a_{ni}^r|X_{ni}|^r \le \varepsilon) + C\varepsilon^{2/r} \sum_{i=u_n}^{v_n} P(a_{ni}^r|X_{ni}|^r > \varepsilon) \\ &\le C \sum_{i=u_n}^{v_n} a_{ni}^2 E X_{ni}^2 I(a_{ni}^r|X_{ni}|^r \le \varepsilon^2) + C \sum_{i=u_n}^{v_n} a_{ni}^2 E X_{ni}^2 I(\varepsilon^2 < a_{ni}^r|X_{ni}|^r \le \varepsilon) \\ &\quad + C\varepsilon^{2/r-1} \sum_{i=u_n}^{v_n} a_{ni}^r E|X_{ni}|^r I(a_{ni}^r|X_{ni}|^r > \varepsilon) \\ &=: I_8 + I_9 + I_{10}. \end{aligned}$$

By $r < 2$ and (ii), we get $I_{10} \to 0$ as $n \to \infty$. For I_8, we have

$$\begin{aligned} I_8 &\le C\,\varepsilon^{4/r-2} \sum_{i=u_n}^{v_n} a_{ni}^r E|X_{ni}|^r I(a_{ni}^r|X_{ni}|^r \le \varepsilon^2) \\ &\le C\,\varepsilon^{4/r-2} \sup_{n\ge 1} \sum_{i=u_n}^{v_n} a_{ni}^r E|X_{ni}|^r. \end{aligned}$$

By $r < 2$ and (ii), we have

$$\begin{aligned} I_9 &\le C\,\varepsilon^{2/r-1} \sum_{i=u_n}^{v_n} a_{ni}^r E|X_{ni}|^r I(\varepsilon^2 < a_{ni}^r|X_{ni}|^r \le \varepsilon) \\ &\le C\,\varepsilon^{2/r-1} \sum_{i=u_n}^{v_n} a_{ni}^r E|X_{ni}|^r I(a_{ni}^r|X_{ni}|^r > \varepsilon^2) \to 0 \quad \text{as} \ \ n \to \infty. \end{aligned}$$

Therefore,

$$\lim_{n\to\infty} \sup E\left|\sum_{i=u_n}^{v_n} a_{ni}(X_{ni} - EX_{ni})\right|^r \ \le \ C\,\varepsilon^{2-r}\left(\sup_{n\ge 1} \sum_{i=u_n}^{v_n} a_{ni}^r E|X_{ni}|^r\right)^{r/2}.$$

Since $0 < \varepsilon < 1$ is arbitrary, by $r < 2$ and (i), the proof is completed.

Proof of Theorem B Let $a = n^{(1-r/2)/4}$ and

$$\begin{aligned} Y_i &= -aI(X_i < -a) + X_i I(|X_i| \le a) + aI(X_i > a), \\ Z_i &= X_i - Y_i \ = \ (X_i + a)I(X_i < -a) + (X_i - a)I(X_i > a). \end{aligned}$$

By Lemma 2.1, $\{Y_i, i \geq 1\}$ and $\{Z_i, i \geq 1\}$ are sequences of pairwise NQD. By Theorem 2.1 and Lemma 2.2, we have

$$\begin{aligned}
E\left|n^{-1/r}\sum_{i=1}^{n}X_i\right|^r &\leq Cn^{-1}\left\{E\left|\sum_{i=1}^{n}(Z_i-EZ_i)\right|^r + E\left|\sum_{i=1}^{n}(Y_i-EY_i)\right|^r\right\} \\
&\leq Cn^{-1}E\left|\sum_{i=1}^{n}(Z_i-EZ_i)\right|^r + Cn^{-1}\left\{E\left|\sum_{i=1}^{n}(Y_i-EY_i)\right|^2\right\}^{r/2} \\
&\leq Cn^{-1}\sum_{i=1}^{n}E|Z_i|^r + Cn^{-1}\left\{\sum_{i=1}^{n}EY_i^2\right\}^{r/2} \\
&=: I_{11} + I_{12}.
\end{aligned}$$

By $|Z_i| \leq |X_i|I(|X_i| > a)$ and (1.3), we have

$$I_{11} \ \leq \ C\sup_{n\geq 1} n^{-1}\sum_{i=1}^{n}E|X_i|^r I(|X_i| > a) \to 0 \quad \text{as} \ \ n \to \infty.$$

By $|Y_i| \leq a = n^{(1-r/2)/4}$ and $r < 2$, we have

$$I_{12} \ \leq \ Cn^{-1}\left\{\sum_{i=1}^{n}a^2\right\}^{r/2} = Cn^{(2-r)(r-4)/8} \to 0 \quad \text{as} \ \ n \to \infty.$$

The proof is completed.

References

[1] Lehmann E L. Some concepts of dependence [J]. Ann. Math. Stat., 1966, 37(5): 1137–1153.

[2] Ebrahimi N, Ghosh M. Multivariate negative dependence [J]. Commun. Stat., Theory Methods, 1981, 10(4): 307–337.

[3] Joag-Dev K, Proschan F. Negative association of random variables with applications [J]. Ann. Stat., Theory Methods, 1981, 11(1): 286–295.

[4] Newman C M. Asymptotic independence and limit theorems for positively and negatively dependent random variables[A]. In: Tong, Y.L. (Ed.) Inequalities in Statistics and Probability[C]. IMS Lecture Notes Monogr. Ser. 5, 1984: 127–140.

[5] Matula P. A note on the almost sure convergence of sums of negatively dependent random variables [J]. Statist. Probab. Lett., 1992, 15(3): 209–213.

[6] Wu Q Y. Convergence properties of pairwise NQD random sequences [J]. Acta Math. Sin., Engl. Ser., 2002, 45(3): 617–624 (in Chinese).

[7] Liang H Y, Chen Z J, Su C. Convergence of Jamison-type weighted sums of pairwise negatively quadrant dependent random variables [J]. Acta Math. Appl. Sin., Engl. Ser., 2002, 18(1): 161–168.

[8] Cabrera M O, Volodin A. Mean convergence theorems and weak laws of large numbers for weighted sums of random variables under a condition of weighted integrability [J]. J. Math. Anal. Appl., 2005, 305(2): 644–658.

[9] Wan C G. Law of large numbers and complete convergence for pairwise NQD random sequences [J]. Acta Math. Appl. Sin., 2005, 28(2): 253–261 (in Chinese).

[10] Chen P Y. L^r convergence for pairwise NQD random variables [J]. Acta Math. Sci., Ser. A, 2008, 28(3): 447–453 (in Chinese).

[11] Li R, Yang W G. Strong convergence of pairwise NQD random sequences [J]. J. Math. Anal. Appl., 2008, 344(2): 741–747.

[12] Baek J I, Ko M H, Kim T S. On the complete convergence for weighted sums of dependent random variables under condition of weighted integrability [J]. J. Korean Math. Soc., 2008, 45(4): 1101–1111.

[13] Gan S X, Chen P Y. Some limit theorems for sequences of pairwise NQD random variables [J]. Acta Math. Sci., Ser. B, 2008, 28(2): 269–281.

[14] Meng Y J, Lin Z Y. On the weak laws of large numbers for arrays of random variables [J]. Statist. Probab. Lett., 2009, 79(23): 2405–2414.

[15] Baek J I, Park S T. Convergence of weighted sums for arrays of negatively dependent random variables and its applications [J]. J. Stat. Plann. Inference, 2009, 140(9): 2461–2469.

[16] Gan S X, Chen P Y. Some remarks for sequences of pairwise NQD random variables [J]. Wuhan Univ. J. Nat. Sci., 2010, 15(6): 467–470.

[17] Wu Q Y, Jiang Y Y. The strong law of large numbers for pairwise NQD random variables [J]. J. Syst. Sci. Complex., 2011, 24(2): 347–357.

[18] Wu Y F, Guan M. Mean convergence theorems and weak laws of large numbers for weighted sums of dependent random variables [J]. J. Math. Anal. Appl., 2011, 377(2): 613–623.

[19] Wu Y F, Wang D C. Convergence properties for arrays of rowwise pairwise negatively quadrant dependent random variables [J]. Appl. Math., Praha, 2012, 57(5): 463–476.

[20] Sung H S. Convergence in r-mean of weighted sums of NQD random variables [J]. Appl. Math. Lett., 2013, 26(1): 18–24.

[21] Shao Q M. A comparison theorem on maximum inequalities between negatively associated and independent random variables [J]. J. Theor. Probab., 2000, 13(2): 343–356.

[22] Asadian N, Fakoor V, Bozorgnia A. Rosenthal's type inequalities for negatively orthant dependent random variables [J]. J. Iran. Stat. Soc., 2006, 5(1-2): 66–75.

两两NQD列的Marcinkiewicz-Zygmund不等式及其应用

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摘要: 本文研究了两两NQD随机变量的Marcinkiewicz-Zygmund不等式及其应用的问题. 利用截尾的方法, 获得了两两NQD随机变量的p阶($1 \le p < 2$)Marcinkiewicz-Zygmund不等式结果. 作为应用, 获得了两两NQD随机变量的两个L^r收敛性结果的简单证明, 改进了陈平炎[10]和Sung [20]的相应工作.

关键词: 两两NQD随机变量; Marcinkiewicz-Zygmund不等式; L^r收敛性

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