To decomposize (2) continusely, we obtain

$$\begin{cases} C_\mu^m \sum_{j\in I_i} q_{i,j,1,0}\alpha_{i,j}^{\mu-m}\beta_{i,j}^m - p_{i,m} = r_{i,m}(y-y_i), \\ C_\mu^m \sum_{j\in I_i} q_{i,j,0,1}\alpha_{i,j}^{\mu-m}\beta_{i,j}^m + p_{i,m+1} = r_{i,m}(x-x_i), \\ \sum_{j\in I_i} q_{i,j}(v_i)\alpha_{i,j}^{\mu-m}\beta_{i,j}^m = 0, 0\le m\le \mu, 1\le i\le \theta_0, \end{cases} \tag{3}$$

where $r_{i,m}, 0\le m\le\mu$ are also in $\mathcal{F}$ and $q_{i,j,1,0}, q_{i,j,0,1}\in\mathcal{F}$ such that $q_{i,j}=q_{i,j,1,0}(x-x_i)+q_{i,j,0,1}(y-y_i)+q_{i,j}(v_i)$. (3) is called the second decomposition of (1). Using the same skill, we can derive farther decompositions of (1). In pasticular, we can choose that

$$q_{i,j,a,b}=\int_0^1 \frac{\partial^{a+b}}{\partial x^a\partial y^b} q_{i,j}(x_i+t(x-x_i), y_i+t(y-y_i))dt. \tag{4}$$

and obtain the following interesting.

Theorem 2 *To choose $q_{i,j,1,0}$ and $q_{i,j,0,1}$ as (4), from (3) we know that the solutions of (1) are some solutions of the system of linear integral differential equations.*

研究多元样条的逐次分解法

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摘　要

本文在协调方程的基础上提出了研究多元样条的逐次分解法，并由此证明了多元样条(包括多项式样条、有理样条乃至更一般的样条)在本质上是一个积分微分方程组的解. 该方法具有以下优点: 1) 即可研究多项式样条，又可以研究有理样条乃至更一般的样条; 2) 即适用于三角剖分，又适用于直线剖分乃至更一般的代数曲线剖分; 3) 即能用于研究样条空间，又能用于研究样条环; 4) 可使许多问题局部化.

Journal of Mathematical Research & Exposition
Vol.14, No.2, May 1994

Decomposition Method for Studying Multivaliate Splines *

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In this note we present, basing on the conformality conditions, a so-called decomposition method to study multivaliate splines. By this method it is proved that multivaliate splines (including both polynomial splines and rational splines and even more general splines) are in essential the solutions of a system of linear integral differential equations. This method has also the following advantages: 1) it can be used to study both polynomial splines and rational splines and even more general splines, 2) it is efficient for splines on both triangulations and rectilinear partitions and even on algebrica curve partitions, 3) it is suitable to study both spline spaces and spline rings, and 4) it can be used to localize many problems. Let $\Omega \subset R^2$ a simply connected domain and $\Delta = \{\Omega_i; 1 \le i \le \delta\}$ be an its rectilinear partition, i.e., for each $i, \partial\Omega_i$ is homeomorphic to a circle and $\partial\Omega_i \cap \Omega$ is a piecewise linear curve, and let $v_i = (x_i, y_i), 1 \le i \le \theta$ be the vectices of Δ and $v_i, 1 \le i \le \theta_0$ be its inner vertices. We denote by $I_i = \{j; v_j$ is adjacent to $v_i\}$ and $l_{i,j} = (x_j - x_i)(y - y_i) - (y_j - y_i)(x - x_i)$ as well. It is well-known that to study spline space $S_k^{\mu-1}(\Delta)$ one needs only to study conformality conditions

$$\sum_{j\in I_i} q_{i,j} l_{i,j}^{\mu} = 0, \quad 1 \le i \le \theta_0, \tag{1}$$

where $q_{j,i} = (-1)^{\mu+1} q_{i,j}$ are called smothness cofactors. A function set $\mathcal{F}$ is called with i-Bezout property if for an algebrica curve l of degree i which is fixed by $p = 0$ and for an $f \in \mathcal{F}$ satistying $f|_l = 0$, it holds $f = f_1 p$ for an $f_1 \in \mathcal{F}$.

Theorem 1 *Let $\mathcal{F}$ be a function set with 1-Bezout property, and $S^{\mu-1}(\Delta) = \{f \in C^{\mu-1}(\Omega); f|_{\Omega_i} \in \mathcal{F}, \forall \Omega_i \in \Delta\}$. Then (1) is equivalent to*

$$\begin{aligned} C_\mu^m \sum_{j\in I_i} \alpha_{i,j}^{\mu-m} \beta_{i,j}^{\mu} q_{i,j} &= -p_{i,m+1}(y - y_i) + p_{i,m}(x - x_i), \\ &\quad 0 \le m \le \mu, 1 \le i \le \theta_0, \end{aligned} \tag{2}$$

where $p_{i,m}, 1 \le m \le \mu$ are some functions in $\mathcal{F}$ and $p_{i,0} = p_{i,\mu+1} \equiv 0$, and $\alpha_{i,j} = y_i - y_j$ and $\beta_{i,j} = x_j - x_i$. (2) is called the first decomposition of (1).

*Received Feb. 28, 1994.

To decompozize (2) continusely, we obtain

$$\begin{cases} C_\mu^m \sum_{j\in I_i} q_{i,j,1,0}\alpha_{i,j}^{\mu-m}\beta_{i,j}^m - p_{i,m} = r_{i,m}(y-y_i), \\ C_\mu^m \sum_{j\in I_i} q_{i,j,0,1}\alpha_{i,j}^{\mu-m}\beta_{i,j}^m + p_{i,m+1} = r_{i,m}(x-x_i), \\ \sum_{j\in I_i} q_{i,j}(v_i)\alpha_{i,j}^{\mu-m}\beta_{i,j}^m = 0, 0\le m\le \mu, 1\le i\le \theta_0, \end{cases} \tag{3}$$

where $r_{i,m}, 0\le m\le\mu$ are also in $\mathcal{F}$ and $q_{i,j,1,0}, q_{i,j,0,1}\in\mathcal{F}$ such that $q_{i,j}=q_{i,j,1,0}(x-x_i)+q_{i,j,0,1}(y-y_i)+q_{i,j}(v_i)$. (3) is called the second decomposition of (1). Using the same skill, we can derive farther decompositions of (1). In pasticular, we can choose that

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