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# Elimination in Weyl Algebra and Identities *

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In paper [1], Zeilberger presented an approach to special functions identities, besides providing a general mathematical framework for the proof machinery, which leads to mathematically and algorithmically challenging questions such as elimination in noncommutative operator algebras (Weyl algebra). By the impetus of this approach, we study the elimination under the shift operators.

Let us define the shift operators $N$ and $K$ by $Nf(n,k) := f(n+1,k), Kf(n,k) := f(n,k+1)$. A partial recurrence operator with polynomial coefficients is any polynomial in the four indeterminates $n,k,N,K$. They satisfy the commutation relations: $Nn = (n+1)N, Nk = (k+1)K, NK = KN, Nk = kN, nK = Kn, nk = kn$.

For simplicity, we just give the main theorem in Weyl algebra $C\langle N,n\rangle$.

**Theorem** *For any given polynomials $A, B \in C\langle N,n\rangle$, there always are polynomials $u, v \in C\langle N,n\rangle$, such that $uA = vB$.*

By using the elimination in Weyl algebra, we designed an algorithm GETUV and wrote programs in MAPLE to prove hypergeometric identities of the form $\sum_k F(n,k) = rhs(n)$, where $F(n,k)$ and $rhs(n)$ should be quotients of products of factorials of the form $(an + bk + c)!$. We have already proved identities in paper [1], including Saalschutz's identity, Vandermonde-Chu convolution formula, and Dixon identity, and 196 identities in the book[2].

See the following table.

| CPU time | $< 0.05s$ | $0.05s - 0.1s$ | $0.1s - 0.5s$ | $0.5s - 1s$ | $1s - 3s$ | $> 3s$ |
|---|---|---|---|---|---|---|
| number | 133 | 16 | 22 | 14 | 6 | 5 |
| proportion | 67.875% | 8.163% | 11.224% | 7.143% | 3.061% | 2.551% |

The whole paper will appear in some journal later.

## References:

[1] ZEILBERGER D. *A holonomic systems approach to special functions identities* [J]. Journal of Computational and Applied Mathematics, 1990, **32**: 321–368.
[2] GOULD H W. *Combinatorial Identities* [M]. Morgantown Printing Company, 1972.

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