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(α, β)-Inversion Formulas

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Recently, basing on a suitable application of Milne-Bhatnagar's characterization theorem about matrix inversions, we have found that Warnaar's elliptic matrix inversion can be further extended to the following general inversion theorem.

Theorem *Let $\{\alpha_{ij}\}$ and $\{\beta_{ij}\}$ be two arbitary double index sequences over the complex number field* $\mathbf{C}$, *such that none of the terms β_{ij} $(i \neq j)$ and α_{ij} is zero, and that β_{ij} is anti-symmetric, i.e., $\beta_{ij} = -\beta_{ji}$. Assume that $F = (F(n,k))$ and $G = (G(n,k))$ (with $n \geq k \geq 0$) are infinite-dimensional lower-triangular matrices with entries defined by*

$$F(n,k) = (\prod_{i=k}^{n-1} \alpha_{ik})/(\prod_{i=k+1}^{n} \beta_{ik}) \tag{1}$$

$$G(n,k) = (\alpha_{kk} \prod_{i=k+1}^{n} \alpha_{in})/(\alpha_{nn} \prod_{i=k}^{n-1} \beta_{in}) \tag{2}$$

where any product over an empty set is 1. Then, F and G are inverse matrices if and only if for integers p, q, n, k

$$\beta_{p,q}\alpha_{n,k} - \beta_{p,k}\alpha_{n,q} + \beta_{q,k}\alpha_{n,p} = 0. \tag{3}$$

This general result contains various well-known inversion pairs as special cases, e.g. those of Krattenthaler's (including both Gould-Hsu's and Calitz's), Gasper's, Warnaar's and Ma's extension of Warnarr's, etc. A somewhat complicated proof will be given elsewhere. May we hope that somebody could find some simple/short proof for the result mentioned above?

References:

[1] WARNAAR S O. *Summation and transformation formulas for elliptic hypergeometric series* [J]. Constr. Approx., 2002, **18**: 479–502.

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