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Single Machine Scheduling Problems with General Learning Effect

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Abstract: In a learning effect environment, the machine improves continuously as a result of repeating the same or similar jobs, hence the later a given job is scheduled in the sequence, the shorter its processing time is. In this paper, we consider a new general learning effect, i.e. Dejong's learning effect. Using this Dejong's learning effect polynomial solutions for the single machine makespan minimization problem, total flow time minimization problem and two classes of single machine multi-criteria problems are obtained.

Key words: scheduling; single machine; learning effect.
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1. Introduction

Scheduling problems have received considerable attentions for many years. A common assumption in the traditional scheduling is that the production time of a given product is independent of its position in the production sequence. However, in many realistic settings, because firms and employees perform a task over and over, they learn how to perform more efficiently. Many production facilities (a machine, a worker) improve continuously with time. As a result, the production time of a given product is shorter if it is scheduled later, rather than earlier in the sequence. This phenomenon is known as a "learning effect" in the literature. Biskup[1] was the first to investigate the effect of learning in the framework of scheduling. He assumed a learning process that is reflected in a decrease in production time as a function of the number of repetitions of the production of a single item, i.e. as a function of the job position in the sequence. Biskup studied the single machine problem of minimum total flow time, and single machine problem of minimizing the weighted sum of completion time deviations from a common due date and the sum of job completion times. Using similar solution techniques, Mosheiov[2,3] investigated several other single machine problems, and the identical parallel machines scheduling problem of minimum total flow time.

In this paper we study a general learning effect, i.e. Dejong's learning effect. We show that the minimum total flow time problem is solved by the shortest (normal) processing time first (SPT) schedule. We prove that SPT rule also solves the makespan minimization problem with a Dejong's learning effect (unlike the classical version in which the makespan value is sequence-independent). We then solve two classes of single machine multi-criteria problems (i.e. we seek

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a schedule that performs well with respect to several measures).

2. Assumptions

The single machine scheduling consists of n jobs on one machine. n jobs are available at time zero. Each job has a normal processing time and the jobs are indexed according to the shortest (normal) processing time (SPT) rule, i.e., $p_1 \le p_2 \le \cdots \le p_n$. The normal processing time of a job is incurred if the job is scheduled first in a sequence. The processing times of the following jobs are smaller than their normal processing times because of the learning effect. For a given schedule π, $C_j = C_j(\pi)$ represents the completion time of job j. The previous work reported above adopts the well-known " unit cost " learning model, in which the processing time of job j if it is scheduled in position r is $p_{jr} = p_j r^a$ where $a \le 0$ is a constant learning index. In fact, many different learning curve models have been proposed and used, for a review see Badiru [4]. Thus, we shall adopt a new type of learning curve model, i.e., Dejong's learning curve[5]. Let p_{jr} if scheduled in position r, be given by

$$p_{jr} = p_j[M + (1-M)r^a], j, r = 1, 2, \cdots, n, \tag{1}$$

where $a \le 0$ is a constant learning index, $0 \le M \le 1$. When $M = 1$, it is the classical scheduling problem, when $M = 0$ it is the model of Biskup[1] and Mosheiov[2,3]. Thus, our model is quite general.

For convenience, we denote the Dejong's learning effect given in (1) by DLE. Thus, using the conventional notation for describing scheduling problems, we denote, e.g. the single machine makespan minimization problem by $1|DLE|C_{\max}$. In the following, we examine two classical single machine scheduling problems with the above learning effect.

Theorem 2.1 *For the problem $1|DLE|\sum C_j$, an optimal schedule can be obtained by SPT rule.*

Proof The proof can be established by the standard pairwise job interchange argument. Assume $p_i \le p_j$ and job j is scheduled directly before job i in the rth position in a sequence. Let B and A be the sum of the flowtimes of the jobs scheduled before and after the jobs j and i, respectively and C_{ji} be the overall objective function value. If interchanging the jobs j and i, the resulting new scheduling does not increase the sum of its flowtimes. The value of A does not increase, either. Let C_{ij} be the sum of flowtimes yielded by the new sequence and let t be the completion time of the job occupying the $(r-1)$th position. Then one has

$$C_{ji} = B + (t + p_j(M + (1-M)r^a)) + (t + p_j(M + (1-M)r^a) + p_i(M + (1-M)(r+1)^a)) + A$$

and

$$C_{ij} \le B + (t + p_i(M + (1-M)r^a)) + (t + p_i(M + (1-M)r^a) + p_j(M + (1-M)(r+1)^a)) + A.$$

Hence, we obtain

$$C_{ji} - C_{ij} \ge (p_j - p_i)[M + (1-M)(2r^a - (r+1)^a)] \ge 0$$

for $p_i \le p_j$. Note that $(2r^a - (r+1)^a)$ is non-negative, because $(r+1)^a < r^a$. Repeating this interchange argument for all jobs not sequenced according to SPT rule, the theorem is obtained.

Theorem 2.2 *For the problem* $1|DLE|C_{\max}$, *an optimal schedule can be obtained by SPT rule.*

Proof This proof is similar to the proof process of Theorem 2.1.

3. Multi-criteria problems

In this section, we study two single machine multi-criteria problems with a Dejong's learning effect. Although the computational effort is significantly greater when a Dejong's learning effect is assumed, in both cases the optimal solution procedure remains polynomial.

3.1. A due-date assignment problem[6]

Consider an n-job single machine problem, with p_j denoting the (sequence independent) processing time of job j ($j = 1, 2, \cdots, n$). All jobs share a common due-date d, yet to be determined. For a given schedule π, $C_j = C_j(\pi)$ represents the completion time of job j, $E_j = \max\{0, d - C_j\}$ is the earliness value of job j, and $T_j = \max\{0, C_j - d\}$ is the tardiness value of job j, $j = 1, 2, \cdots, n$. Further, let α, β and γ be the per time unit penalties for due-date delay, earliness and tardiness. The general objective is to find a schedule π and d which minimizes

$$f(d, \pi) = \sum_{i=1}^{n} (\alpha d + \beta E_i + \gamma T_i). \tag{2}$$

Panwalker et al.[6] showed that (i) for any given schedule, it is optimal to assign the due-date at the completion time of the kth job, where

$$k = \lceil n(\gamma - \alpha)/(\beta + \gamma) \rceil, \tag{3}$$

and (ii) the optimal schedule is V-shaped, i.e. early jobs are arranged in a non-increasing order of processing times, and tardy jobs are arranged in a non-decreasing order of processing times. The positional weight of position r in the sequence is given by

$$\omega_r = \begin{cases} n\alpha + (r-1)\beta & \text{if} \quad r \le k, \\ (n+1-r)\gamma & \text{if} \quad r > k. \end{cases} \tag{4}$$

Thus, the optimal schedule is obtained by the well-known matching procedure of the largest processing time to the smallest positional weight, the next larger processing time to the next smaller positional weight, etc.

Now assume a Dejong's learning effect as given in (1). In this new setting, our objective remains to find d and π that minimize (2). Let x_{jr} be a 0/1 variable such that $x_{jr} = 1$ if job j is scheduled in position r, and $x_{jr} = 0$, otherwise. As in Mosheiov[2], the optimal matching of jobs to positions requires a solution for the following assignment problem:

$$\min \sum_{j=1}^{n} \sum_{r=1}^{n} \omega_r p_{jr} x_{jr} \tag{5}$$

subject to

$$\sum_{j=1}^{n} x_{jr} = 1, \quad r = 1, 2, \cdots, n,$$

$$\sum_{r=1}^{n} x_{jr} = 1, \quad j = 1, 2, \cdots, n,$$

$$x_{jr} = 0 \text{ or } 1, \quad j, r = 1, 2, \cdots, n.$$

Recall that solving an assignment problem of size n requires an effort of $O(n^3)$. Once the assignment problem is solved, we obtain an optimal matching of jobs to positions (i.e. an optimal sequence). Note that the value of k[3] is independent of the actual processing times of the jobs and, hence, it is not affected by the Dejong's learning effect[1,2]. Thus, the optimal due-date is at the completion time of the kth job in the sequence obtained from the solution of the assignment problem.

In order to demonstrate the above, we solve the instance introduced (and solved for the case of no learning effect) by Panwalker et al.[6] with a Dejong's learning effect assumption:

Example 1[6] Data: $n = 7, p_1 = 3, p_2 = 4, p_3 = 6, p_4 = 9, p_5 = 14, p_6 = 18, p_7 = 20, \alpha = 5, \beta = 11, \gamma = 18$ and $M = 0.4$. The optimal k-value is 4. The positional weights are: $\omega_1 = 35, \omega_2 = 46, \omega_3 = 57, \omega_4 = 68, \omega_5 = 54, \omega_6 = 36, \omega_7 = 18$. The optimal sequence (with no learning effect) is $(6, 4, 2, 1, 3, 5, 7)$. Thus the optimal due-date is at time $p_6 + p_4 + p_2 + p_1 = 34$, and the total cost is 2664. Assume now a Dejong's learning curve, for example: $a = -0.5$. The input for the assignment problem (5), i.e., job/location processing times and positional weights, are given in Table 1. The solution of the assignment problem leads to a new optimal sequence: $(5, 3, 2, 1, 4, 6, 7)$. The optimal k-value remains 4. The optimal due-date is at $14.00 + 4.95 + 3.00 + 2.10 = 24.05$, and the total cost is 2000.3. The optimal sequence is clearly different from the optimal sequence in the original version of the problem.

Table 1. Job/location processing times and positional weights for the due-date assignment problem of Panwalker et al. [6][a]

j	r						
	1	2	3	4	5	6	7
1	3.00	2.47	2.24	**2.10**	2.01	1.93	1.88
2	4.00	3.30	**3.00**	2.80	2.67	2.58	2.51
3	6.00	**4.95**	4.48	4.20	4.01	3.87	3.76
4	9.00	7.42	6.72	6.30	**6.02**	5.80	5.64
5	**14.00**	11.54	10.45	9.80	9.36	9.03	8.77
6	18.00	14.84	13.44	12.60	12.03	**11.61**	11.28
7	20.00	16.49	14.93	14.00	13.37	12.90	**12.54**
ω_r	35	46	57	68	54	36	18

[a]The optimal sequence (5,3,2,1,4,6,7) (see bold numbers) is obtained by solving the associated assignment problem. j-job index; r-location index, ω_r-positional weight.

3.2. Simultaneous minimization of the total completion time and variation of completion times[7]

In this single machine bi-criteria problem, we look for a schedule that performs well with respect to both a classical efficiency measure (total completion time) and a measure of performance balance (variation of completion times). As in section 2, p_j denotes the (sequence-independent) processing time of job j, and (for a given schedule) C_j represents the completion time of job j,

$j=1,2,\cdots,n$. TC denotes the total completion time TC=$\sum_{j=1}^n C_j$. TADC denotes the total absolute differences in completion times TADC= $\sum_{i=1}^n \sum_{j=1}^n |C_i - C_j|$. Let $0 \le \delta \le 1$. The objective is to find a schedule that minimizes the linear combination of both measures:

$$f(\pi) = \delta \mathrm{TC} + (1-\delta)\mathrm{TADC}. \tag{6}$$

Bagchi[7] showed that the positional weight of position r in the sequence is given by

$$\omega_r = (2\delta - 1)(n+1) + r[2 - 3\delta + n(1-\delta)] - r^2(1-\delta), r = 1,2,\ldots,n. \tag{7}$$

The optimal schedule is obtained, as in section 3.1, by the same matching procedure of jobs to positions.

When the learning effect specified in (1) is assumed, we have to solve the assignment problem (5) (with the appropriate positional weights).

4. Conclusions

We study a new type of learning effect, i.e. a Dejong's learning effect. First, we give out polynomial solutions ($O(n\log n)$) for the single machine makespan minimization problem and total flow time minimization problem. Then, we solve two classes of single machine multi-criteria problems that can be formulated as assignment problems and hence these are also solved polynomially.

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具有一般学习效应的单机排序问题

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摘要: 在具有学习效应的环境下, 由于机器重复加工相同或相似的工件, 因此以后加工的工件的加工时间变小. 本文研究新的更一般的学习效应: Dejong 学习效应. 我们证明单机最大完工时间问题, 总完工时间问题和两类多目标问题是多项式时间可解的.

关键词: 排序; 单机; 学习效应.