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Pointwise Pseudo-Orbit Tracing Property and Its Application

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Abstract: In this paper, the pointwise pseudo-orbit tracing property is defined on a compact metric space, and it is a generalization of pseudo-orbit tracing property. As applications, we prove the following results: (i) If f has pointwise pseudo-orbit tracing property, for any $k \in Z_+$, and f^k is chain transitive, then for any $k \in Z_+$, f^k has open set transitive ; (ii) If f has pointwise pseudo-orbit tracing property, and for any $n \in Z_+, f^n$ is chain transitive, then f has sensitive dependence on initial conditions; (iii) If f is open set mixing and has pointwise pseudo-orbit tracing property, then f has property P; (iv) Let $f : (X,d) \to (X,d)$ be a homeomophism, then f is pointwise pseudo-orbit tracing property if and only if the shift map σ_f is pointwise pseudo-orbit tracing property.

Key words: sensitive dependence; pseudo-orbit tracing property.
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1. Introduction

From now on, we will always suppose that (X,d) is a compact metric space and $f : X \to X$ is a continuous map.

By using three conditions as the essential features of chaos, Devaney established a chaotic dynamical systems [1-4]. Let σ_f denote the shift map of the inverse limit spaces. The concept of pseudo-orbit has firstly appeared in the work of Anosov[5-6], and is closely related with the property of stability[7]. Chen and Li defined the asymptotically shadowing property and proved that asymptotically shadowing property and shadowing property are equivalent if f is continuous and onto[8]. A general discussion on the dynamical properties of the inverse limit spaces is contained in [9]. Mai investigated pointwise recuuent dynamical systems with pseudo-orbit tracing property, and proved that each pointwise recurrent C^0-flow on a chain-connected space having the pseudo-orbit tracing property must be a minimal flow, i.e., the whole space must be a (unique) minimal set of F[10].

In this paper, the pointwise pseudo-orbit tracing property is defined on a compact metric space, and it is a generalization of pseudo-orbit tracing property. As applications, we prove the following results: (i) If f has pointwise pseudo-orbit tracing property, and for any $k \in Z_+, f^k$

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is chain transitive, then for any $k \in Z_+$, f^k has open set transitive ; (ii) If f has pointwise pseudo-orbit tracing property, and for any $n \in Z_+$, f^n is chain transitive, then f has sensitive dependence on initial conditions. (iii) If f is open set mixing and has pointwise pseudo-orbit tracing property, then f has property P; (iv) Let $f:(X,d)\to(X,d)$ be a homeomophism, then f is pointwise pseudo-orbit tracing property if and only if σ_f is pointwise pseudo-orbit tracing property.

2. Pointwise pseudo-orbot tracing property

Let Z_+ denote the positive integer number set, and $\mathrm{Orb}_f(x)=\{f^n(x)|n\geq 0\}$ denote the orbit of f in x.

Definition 2.1 *Let $f:X\to X$ be a continuous map. We say f has pointwise pseudo-orbit tracing property if for any $\varepsilon>0$ there exists a real number $\delta>0$ such that for any δ-pseudo-orbit $(x_0,x_1,\cdots)$, $(x_N,x_{N+1},\cdots)$ is ε-traced for some integer $N\geq 0$.*

Lemma 2.2 *If f has pointwise pseudo-orbit tracing property, then for any $k\in Z_+, f^k$ has also pointwise pseudo-orbit tracing property.*

By the definition, the tracing property of f implies the asymptotically tracing property of f, and the asymptotically tracing property of f implies the pointwise pseudo-orbit tracing property of f.

Definition 2.3 *A continuous map $f:X\to X$ is said to be open set transitive if for any nonempty open sets U,V, there exists $n\geq 1$ with $f^n(U)\bigcap V\neq\emptyset$. f is said to be open set mixing if for any two nonempty open sets U and V in X, there exists $N\in Z_+$, such that $f^k(U)\bigcap V\neq\emptyset$ for any $k\geq N$.*

We say $(x_0,x_1,\cdots,x_n)$ is an ε-chain from x to y if $x,y\in X,\varepsilon>0$, there exists a sequence $(x_0,x_1,\cdots,x_n)$ such that $d(x_{i+1},f(x_i)\leq\varepsilon, 0\leq i\leq n-1, x_1=x, x_n=y$. $n+1$ is called the length of the ε-chain.

Definition 2.4 *f is said to be chain transitive if for any $\varepsilon>0,\forall x,y\in X$, there exists an ε-chain from x to y.*

Theorem 2.5 *If f has pointwise pseudo-orbot tracing property, $k\in Z_+, f^k$ is chain transitive, then f^k is open set transitive.*

Proof For any $x,y\in X$, let $B(x,\varepsilon_1)=\{z\in X:d(x,z)<\varepsilon_1\}$, $B(y,\varepsilon_2)=\{z\in X:d(y,z)<\varepsilon_2\}$, respectively. Assume that f has pointwise pseudo-orbit tracing property. By using Lemma 2.2, for any $k\in Z_+, f^k$ has also pointwise pseudo-orbit tracing property. Therefore, for any $0<\varepsilon<\min\{\varepsilon_1,\varepsilon_2\}$, there exists $\delta>0$ such that for any δ-pseudo-orbit $(x_0,x_1,\cdots)$ of f^k, there exist an integer $N\in Z_+$ and a point $z\in X$ such that $(x_N,x_{N+1},\cdots)$ is ε-traced by the orbit of f^k on z. As f^k is chain transitive, there exist a δ-chain $\alpha=(y_0=x,y_1,\cdots,y_m=y)$ from x to y and a δ-chain $\beta=(y_m=y,y_{m+1},\cdots,y_n)$ from y to x. Let $\overline{\alpha}=(y_0=x,y_1,\cdots,y_{m-1})$,

$\overline{\beta} = (y_m = y, y_{m+1}, \cdots, y_{n-1})$, and $A = (\overline{\alpha}, \overline{\beta}, \overline{\alpha}, \overline{\beta}, \cdots) = (z_0, z_1, \cdots)$. Then, A is a δ-pseudo-orbit of f^k, and $(z_N, z_{N+1}, \cdots)$ is ε-traced by some point $w \in X$. Then there exist $i < j < Z_+$, such that $z_{N+i} = x$, $z_{N+j} = y$ satisfying:

$$d(f^{ki}(z), z_{N+i}) < \varepsilon,$$

$$d(f^{kj}(z), z_{N+i}) < \varepsilon.$$

So

$$f^{k(j-i)}(B(x, \varepsilon_1)) \cap B(x, \varepsilon_2) \neq \emptyset.$$

Hence, f^k is open set transitive.

3. Sensitive dependence on initial conditions

Definition 3.1 *A continuous map $f : X \to X$ is sensitive on initial conditions if there exist $\delta > 0$ such that for any neighhood U of x, there exist a point $y \in U$ and an integer $n \geq 0$ such that $d(f^n(x), f^n(y)) > \delta$.*

Definition 3.2 *A continuous map $f : X \to X$ is called to be orbit topological transitive if there is some $x \in X$ such that $\{f^n(x)|n \geq 0\} = X$.*

Lemma 3.3[1] *The following statements are equivalent for a continuous map $f : X \to X$ with $fX = X$.*

(i) f is orbit topological transitive.

(ii) Whenever E is a closed subset of X and $E \subset T^{-1}E$ then either $E = X$ or E is nonwhere dense (equivalently whenever U is an open subset of X and $f^{-1}U \subset U$ then $U = \emptyset$ or U is dense).

(iii) Whenever U, V are nonempty open sets, there exist $n \geq 1$ with $f^{-n}U \bigcap V \neq \emptyset$.

(iv) Whenever U, V are nonempty open sets, there exist $n \geq 1$ with $f^n U \bigcap V \neq \emptyset$.

(v) The set of points x with $\{f^n(x) \mid n \geq 0\}$ dense in X is a dense G_δ.

In 1989, Devany proposed the following mathematical definition of chaos[2].

Definition 3.4 *Let f be a continuous map from a metric space X into itself. f is said to be chaotic in the sense of Devenay if*

(1) f is transitve,

(2) the periodic points of f are dense in X , and

(3) f has sensitive dependence on initial conditions.

Sensitive dependence on initial conditions is usually understood to be the main character of chaos. Banks et al. prove that if f is transitive and the periodic points of f are dense in X, then f has sensitive dependence on initial conditions[3]. In the following theorem, we give a sufficient condition for sensitive dependence on initial conditions for f with pointwise pseudo-orbit tracing property.

Theorem 3.5 *Suppose X is a compact metric space, f has pointwise pseudo-orbit tracing* *property, and for any $n \in Z_+$, f^n is chain transitive. Then, f has sensitive dependence on initial conditions.*

Proof For any $n \in Z_+$, f^n is chain transitive, then X is a space with infinite points or with a single point. If X is a space with a single point, then we have our result. In the following, we suppose X is a space with infinite points. Then there exist $\varepsilon > 0$ and $x \in X$, such that $X \setminus B(x, \varepsilon)$ has infinite points.

If f has no sensitive dependence on initial conditions, then there exists a stable point $y \in X$. That is to say, there exists $0 < \delta_1 < \frac{\varepsilon}{6}$ such that for any $n \in Z_+$, $w \in X$, if $d(w, y) < \delta_1$, then $d(f^n(x), f^n(y)) < \frac{\varepsilon}{6}$. As f has pointwise pseudo-orbit tracing property, then there exists $0 < \delta_2 < \delta_1$ such that for any δ_2-pseudo-orbit $(x_0, x_1, \cdots)$ of f then $(x_N, x_{N+1}, \cdots)$ is δ_1-traced by the orbit of a points $w \in X$ for some integer $N \in Z_+$. By Theorem 2.4, f is open set transitive, then there exists $k \in Z_+$ such that $f^k(B(x, \frac{\delta_2}{2})) \bigcap B(x, \frac{\delta_2}{2}) \neq \emptyset$. Let $z \in B(x, \delta_2)$, and $f^k(z) \in B(x, \frac{\delta_2}{2})$. As f^k is chain transitive, then there exists δ_2-chain $(y = y_0, y_1, \cdots, y_n = z)$ of f^k from y to z and δ_2-chain $(y_n = z, y_{n+1}, \cdots, y_m = y)$ of f^k from z to y. Let $I_i = \{y_i, f(y_i), \cdots, f^{k-1}(y_i)\}$, $0 \leq i < m$, $I = I_0 \cup I_1 \cup \cdots \cup I_{m-1}$, then $T = (I, I, \cdots) = (x_0, x_1, \cdots)$ is a δ_2-pseudo-orbit of f. Therefore, there exist $N \in Z_+$ and $w \in X$ such that $(x_N, x_{N+1}, \cdots)$ is δ_1-traced by the orbit of w.

In fact, we can suppose that $u = f^p(w)$ for some $l_0, p \in Z_+$ such that $d(u, y) < \delta_1$. Then

$$d(f^{(l)k}(y), z) \leq d(f^{(l)k}(y), d(f^{(l)k}(u)) + d(f^{(l)k}(u), z) < \frac{\varepsilon}{6} + \frac{\varepsilon}{6} = \frac{\varepsilon}{3}$$

and

$$d(f^{(l)k}(y), x) \leq d(f^{(l)k}(y), z) + d(x, z) < \frac{\varepsilon}{3} + \frac{\varepsilon}{6} = \frac{5\varepsilon}{12}$$

for all $l \geq l_0$. Therefore, $\overline{\mathrm{Orb}_{f^k}(y)} \setminus B(x, \varepsilon)$ is a finite points set, and $\overline{\mathrm{Orb}_{f^k}(y)} \neq X$. Then there exist $y' \in X$ and $0 < \eta < \delta_1$, such that

$$B(y', \eta) \subset X \setminus \overline{\mathrm{Orb}_{f^k}(y)}. \tag{*}$$

On the other hand, y is a stable point of f. Then, for $\eta > 0$, there exists $0 < \delta < \eta$ such that for any $n \in Z_+$, $w \in X$, if $d(y, w) < \delta$, then $d(f^n(y), f^n(w)) < \frac{\eta}{2}$. By Theorem 2.4, f^k is open set transitive, and by using Lemma 3.3, there exists an u such that $\overline{\mathrm{Orb}_{f^k}(u)} = X$. Therefore, there exists $\delta > \sigma > 0$, such that $d(f^{jk}(u), y) < \sigma$ for some $j \in Z_+$. Then, there exists $i > j > 0$ such that $d(f^{ik}(u), y') < \frac{\eta}{2}$. As $d(f^{ik}(u), f^{(i-j)k}(y)) < \frac{\eta}{2}$, then

$$d(f^{(i-j)k}(y), y') \leq d(f^{(i-j)k}(y), f^{ik}(u)) + d(f^{ik}(u), y') < \frac{\eta}{2} + \frac{\eta}{2} = \eta.$$

It is contradictory to the assuption of $(*)$. The theorem is proved.

4. On property P

Definition 4.1 *We say f has property P, if for any two nonempty open set U_0, U_1 in X, there exists $N \geq 2$, for any finite sequence $\{V_1, \cdots, V_k\}$, there exists $x \in X$ such that $f^{(i-1)N}(x) \in V_i$, where $V_i \in \{U_0, U_1\}$, $1 \leq i \leq k$.*

Generally, property P is relative to the positive entropy system[11]. The following theorem shows the relations between property P and pointwise pseudo-orbit tracing property.

Theorem 4.2 *Suppose X is a metric space and $f: X \to X$ is a continuous map. If f is open set mixing and has pointwise pseudo-orbit tracing property, then f has property P.*

Proof In fact, we can assume that $U_\varepsilon(x_i)$ is any sphere neighborhood with radii ε and center in $x_i, i = 1, 2$. If f has pointwise pseudo-orbit tracing property, then there exists $\delta > 0$ such that any δ-pseudo-orbit $(x_0, x_1, \cdots)$ of f, there exists $l \in Z_+$ such that $(x_l, x_{l+1}, \cdots)$ is $\frac{\varepsilon}{2}$-traced for f. As f is open set mixing, there exists $N > 0$ such that for any $U, V \in \{U_{\varepsilon'}(x_i) : i = 1, 2\}$, we have $f^N(U) \bigcap V \neq \emptyset$, where $\varepsilon' = \min\{\frac{\varepsilon}{2}, \frac{\delta}{2}\}$. For any finite sequence

$$I = \{U_0, U_1, \cdots, U_k, U_{k+1} = U_0\}, U_i \in \{U_\varepsilon(x_j) : j = 1, 2\}, 0 \leq i \leq k.$$

Let

$$I' = \{U'_0, U'_1, \cdots, U'_k, U'_{k+1} = U'_0\}, U'_i \in \{U_{\varepsilon'}(x_j) : j = 1, 2\}, 0 \leq i \leq k,$$

where $U'_i \subset U_i, 1 \leq i \leq k$. Then, $f^N(U'_i) \bigcap U'_{i+1} \neq \emptyset$. Take $x'_i \in U'_i$ such that $f^N(x'_i) \in U'_{i+1}$, $0 \leq i \leq k$. Let

$$T'_i = \{x'_i, f(x'_i), \cdots, f^{N-1}(x'_i)\},$$

and

$$T' = \{T'_0, \cdots, T'_k, T'_{k+1} \cdots\} = \{v_0, v_1, \cdots\}$$

such that $T'_{nk+i+1} = T'_i$, $0 \leq i \leq k, n \in Z_+$. Then, T' is a δ-pseudo-orbit of f. Therefore, there exists $w \in X$ such that $\{v'_l, v'_{l+1}, \cdots\}$ is $\frac{\varepsilon}{2}$-traced by the orbit of w for some $l \in Z_+$. Let $p \geq 0$ such that $v_{l+p} = v_0 = x_0$. Then

$$d(f^{(p+i)}(w), v'_{l+p+i}) = d(f^i(f^p(w)), v'_i) < \frac{\varepsilon}{2}$$

for any $i \geq 0$. On the other hand, $d(v'_{jN}, x_j) = d(x'_j, x_j) < \frac{\varepsilon}{2}$ for any $0 \leq j \leq k$. Then $d(f^{(p+jN)}(w), x_j) < \varepsilon$. Therefore, $f^{(j-1)N}(f^{(N+p)}(w)) \in U_j$ for any $0 \leq j \leq k$. This implies that f has property P.

5. The shift map of the inverse limit space

Let (X, d) be a compact metric space and $f: X \to X$ be continuous.

Definition 5.1 *The inverse limit space $\varprojlim\langle X, f\rangle$ of f is a metric space defined by the sequence*

$$X \xleftarrow{f} X \xleftarrow{f} \cdots X \xleftarrow{f}$$

whose element $\overline{x} = (x_0, x_1, \cdots)$ satisfy $f(x_{i+1}) = x_i$ and the metric is defined by

$$\overline{d}(\overline{x}, \overline{y}) = \sum_{i=0}^{\infty} \frac{d(x_i, y_i)}{2^i}.$$

The shift map $\sum_f : \overset{\lim}{\longleftarrow} \langle X, f\rangle \to \overset{\lim}{\longleftarrow} \langle X, f\rangle$ *is defined by* $\sum_f((x_0, x_1, \cdots)) = (f(x_0), x_0, x_1, \cdots)$ *and the* i*-th projection* π_i *of* $\overset{\lim}{\longleftarrow} \langle X, f\rangle$ *is defined by* $\pi_i(x_0, x_1, \cdots) = x_i$ *for each* $i = 0, 1, \cdots$. *A general discussion on the dynamical properties of the inverse limit spaces is contained in [9].*

Let $M = \sup\{d(x,y) : x, y \in X\}$ be the diameter of X. If f is onto, for any $x_o \in X$, there exist preimages $x_1, x_2, \cdots$ satisfying $f(x_{i+1}) = x_i$ for all $i \geq 0$. By using the definition, $(x_0, x_1, \cdots) \in \overset{\lim}{\longleftarrow} \langle X, f\rangle$. In this case, we will use the notation $(x_0, \cdots, x_k, *)$ to denote the element $(x_0, \cdots, x_k, \cdots)$ of $\overset{\lim}{\longleftarrow} \langle X, f\rangle$.

The following Theorem 5.2 shows the relations between a homeomorphism and its shift map.

Theorem 5.2 *Let* (X, d) *be a compact metric space and* $f : X \to X$ *be continuous. If* f *is homeomorphism, then* f *has pointwise pseudo-orbit tracing property if and only if the shift map* σ_f *has pointwise pseudo-orbit tracing property.*

Proof Suppose that f has pointwise pseudo-orbit tracing property. For any $\varepsilon > 0$, let $m \geq \log(2M/\varepsilon)/\log 2$, then

$$\frac{1}{2^m} \leq \frac{\varepsilon}{2M}.$$

As (X, d) is a compact metric space and $f : X \to X$ is uniform continuous, there exists $0 < \tau < \frac{\varepsilon}{4}$ such that for any $x, y \in X$, if $d(x, y) \leq \tau$, we have

$$d(f^i(x), f^i(y)) \leq \frac{\varepsilon}{4} \tag{*}$$

for any $0 \leq j \leq m$. As f has pointwise pseudo-orbit tracing property, for $\tau > 0$, there exists a real number $\delta > 0$ such that for any δ−pseudo-orbit $(x_0, x_1, \cdots)$, $(x_N, x_{N+1}, \cdots)$ is τ-traced for some integer $N \geq 0$.

Suppose that $\{y(0), y(1), \cdots, y(n), \cdots\}$ is any $\delta/2^m$-pseudo-orbit of σ_f in $\overset{\lim}{\longleftarrow} \langle X, f\rangle$, then

$$\overline{d}(y(n+1), \sigma_f(y(n))) \leq \frac{\delta}{2^m} \tag{**}$$

for any $n \geq 0$. Illustratively we write $y(n) = (y_0^n, y_1^n, \cdots)$, then from $(**)$ we get

$$\frac{d(y_m^{n+1}, y_{m-1}^n)}{2^m} \leq \overline{d}(y(n+1), \sigma_f(y(n))) \leq \frac{\delta}{2^m},$$

or

$$d(y_m^{n+1}, f(y_m^n)) = d(y_m^{n+1}, y_{m-1}^n) \leq \delta.$$

Thus, $\{y_m^0, \cdots, y_m^1, \cdots)$ is a δ-pseudo-orbit of f. By the choice of δ, there exists $x_0 \in X$ such that $\{y_m^N, \cdots, y_m^{N+1}, \cdots)$ is τ-traced by the orbit of x_0. Then

$$d(f^i(x_0), y_m^{N+i}) \leq \tau$$

for any $i \geq 0$. By using of $(*)$, we have

$$d(f^k(f^i(x_0)), f^k(y_m^{N+i})) \leq \frac{\varepsilon}{4}$$

for any $0 \le k \le m$. Let $\overline{z} = (f^m(x_0), f^{m-1}(x_0), \cdots, x_0, *) \in \overleftarrow{\lim} \langle X, f\rangle$. Then

$$\overline{d}(\sum_f^i(\overline{z}), y^{N+i}) \le \sum_{j=0}^{m}(f^{m+i+j}(x_0), y_j^{N+i})/2^j + \frac{\varepsilon}{2}$$
$$\le \sum_{k=0}^{m}(f^k(f^i(x_0)), y_k^{N+i})/2^{m-k} + \frac{\varepsilon}{2} < \frac{2\varepsilon}{4} + \frac{\varepsilon}{2} = \varepsilon.$$

This implies $\{y(N), y(N+1), \cdots\}$ is ε-shadowed by the orbit of $\overline{z}$. That is to say, σ_f has pointwise pseudo-orbit tracing property.

On the other hand, if σ_f has pointwise pseudo-orbit tracing property, then for any $\varepsilon > 0$ there exists a real number $\tau > 0$ such that for any τ-pseudo-orbit $\{y(0), y(1), \cdots\}$, $\{y(N), y(N+1), \cdots\}$ is ε-shadowed for some integer $N \ge 0$. Let $m \in Z_+$ satisfying $m \ge \log(2M/\tau)/\log 2$. Then

$$M/2^m \le \tau/2.$$

As (X, d) is a compact metric space and f is homeomorphism, there exists $\delta > 0$ such that

$$\sum_{i=0}^{m} d(f^{-i}(x_{n+1}), f^{-i+1}(x_n))/2^{m-i} \le \frac{\tau}{4}$$

for any δ-pseudo-orbit $(x_0, x_1, \cdots)$ of f and any $n \ge 0$. Let

$$y(n) = (x_n, f^{-1}(x_n), \cdots, f^{-m}(x_n), \cdots) \in \overleftarrow{\lim} \langle X, f\rangle$$

for any $n \ge 0$. Then

$$\overline{d}(\sigma_f(y(n)), y(n+1)) \le \sum_{i=0}^{m}(f^{-i}(x_{n+1}), f^{-i+1}(x_n))/2^i + \frac{\tau}{2}$$
$$< \frac{2\tau}{4} + \frac{\tau}{2} = \tau.$$

This implies $\{y(0), y(1), \cdots\}$ is τ-pseudo-orbit. Therefore, there exists $\overline{z} = (z_0, z_1, \cdots) \in \overleftarrow{\lim} \langle X, f\rangle$ and $N \in Z_+$ such that

$$\overline{d}(y(N+i), \sigma_f^i(\overline{z})) \le \varepsilon$$

for any $i \ge 0$. That is to say

$$d(y_i^N, f^i(z_0)) \le \varepsilon$$

for any $i \ge 0$. Then, f has pointwise pseudo-orbit tracing property.

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逐点伪轨跟踪性质及其应用

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摘要: 本文给出紧致度量空间逐点伪轨跟踪性质的定义，该定义是伪轨跟踪性质定义的推广. 作为应用，证明如下结论： (i) 若 f 具有逐点伪轨跟踪性质，且对任意 $k \in Z_+$, f^k 为链转换的，那么对任意 $k \in Z_+$, f^k 为开集转换； (ii) 若 f 具有逐点伪轨跟踪性质，且对任意 $n \in Z_+, f^n$ 为链转换的，则 f 具有初始敏感依赖性质； (iii) 若 f 为开集混合的，且具有逐点伪轨跟踪性质，那么 f 具有性质 P; (iv) 设 $f : (X, d) \to (X, d)$ 是同胚映射，那么 f 具有逐点伪轨跟踪性质当且仅当移位映射 σ_f 具有逐点伪轨跟踪性质.

关键词: 敏感依赖； 伪轨跟踪性质.