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Bootstrap Test for Stationarity of Heavy-Tailed Series with Structural Breaks

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Abstract The paper proposes a statistic to test stationarity of series with κ-stable innovations and structural breaks, obtains the asymptotical distribution of the statistic, and proves the consistency of the test. To obtain critic values for the test without the estimation of the index κ, the paper proposes the bootstrap procedures to approximate the distribution, and proves the consistency of the procedures. The simulations demonstrate that the bootstrap test is practical and powerful.

Keywords κ stable innovations; structural breaks; stationarity; Heavy tails; bootstrap.

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1. Introduction

To test the stationarity of series with structural breaks is in the focus of statistics and econometrics. Perron [1, 2] proposed a DF-type statistic to test the stationarity of series with different structural breaks. From then on, a great deal of literature on the test of stationarity arose, such as Perron [3], Banerjee [4], Christiano [6], Zivot [6]. Kim et al. [7] considers the test of unit root in series with changed variances. Bussetti and Harvey [8] proposed a test based on residuals to test the stationarity of series in the case of series with different structural breaks.

Recently, series with infinite varianced innovations arouse the interest of statisticians, such as Athreya[9], Han [10] and Phillips [11]. Just as Guillaume [12] and Mittnik [13], many types of data from economics and finance have the same character: a heavier tail than the normal variants, so it is more precise to model these heavy-tailed data with some κ-stable processes, where the index κ can reflect the heaviness of the data.

However, the test for stationarity of series with κ-stable innovations and structural breaks has attracted little attention. So in this paper, we propose a statistic to test stationarity of series with κ-stable innovations and structural breaks, obtain the asymptotical distribution of the statistic, and prove the consistency of the test. To obtain critic values for the test without

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the estimation of the index κ, the paper proposes the bootstrap procedures to approximate the distribution, and proves the consistency of the procedures.

The rest of this paper is organized as follows. In Section 2, we state the model and necessary assumptions and, describe the bootstrap procedures. The main results will appear in Section 3. Simulation will appear in Section 4.

2. Model and assumptions

We consider the model

$$y_t = \mu_t + \delta\omega_t + \varepsilon_t, \quad t = 1, \ldots, T, \tag{1}$$

where $\mu_t = \mu_{t-1} + \eta_t$, $\{\varepsilon_t, t \geq 1\}$ are independent of $\{\eta_t, t \geq 1\}$, $\{\varepsilon_t, t \geq 1\}$ and $\{\eta_t, t \geq 1\}$ are 0-meaned series in the domain of attraction of the same stable law with $1 < \kappa < 2$, $\omega_t = 1$ for $t > [T\lambda]$, otherwise 0. In the model above, the intercept μ_t of $\{y_t\}$ has a known change at $T_0 = [T\lambda]$.

The null Hypothesis and the alternative one are: H_0: $\mu_t = c$, H_1: $\{\mu_t\}$ is a random walk.

In order to test the hypothesis, we employ the Busetti's statistic based on the regression residuals of $\{y_t\}$ on a constant:

$$\xi_T(\lambda) = \frac{\frac{1}{T^2}\sum_{t=1}^{T}\{\sum_{s=1}^{t} e_s\}^2}{\frac{1}{T}\sum_{s=1}^{T} e_s^2},$$

where $\{e_s, 1 \leq s \leq [T\lambda]\}$ are regression residuals of $\{y_s, 1 \leq s \leq [T\lambda]\}$ on a constant, $\{e_s, [T\lambda] + 1 \leq s \leq T\}$ are residuals of $\{y_s, [T\lambda] + 1 \leq s \leq T\}$ on a constant. When $\{\varepsilon_t\}$ and $\{\eta_t\}$ are normal sequences, namely the index κ=2, the statistic above was used by Busetti [8]. But when the index $1 < \kappa < 2$, variances of $\{\varepsilon_t\}$ and $\{\eta_t\}$ are infinite, we just use the denominator above to obtain a rate without the index.

We can anticipate the asymptotic distribution is a function of stable process. The critic values need the knowledge of the index κ, where the κ is difficult to estimate. We propose the Bootstrap procedures for the test above:

Step 1. Calculate the residuals of $\{y_t\}$ on a constant:

$$\hat{\varepsilon}_t = \begin{cases} y_t - \frac{1}{T\lambda}\sum\limits_{t=1}^{[T\lambda]} y_t, & t \leq [T\lambda], \\ y_t - \frac{1}{T(1-\lambda)}\sum\limits_{t=[T\lambda]+1}^{T} y_t, & t > [T\lambda]; \end{cases}$$

Step 2. For $m \leq T$, select bootstrap samples: $\{\tilde{\varepsilon}_t, 1 \leq t \leq [T\lambda]\}$ from $\{\hat{\varepsilon}_t, 1 \leq t \leq [T\lambda]\}$, $\{\tilde{\varepsilon}_t, [T\lambda] + 1 \leq t \leq T\}$ from $\{\hat{\varepsilon}_t, [T\lambda] + 1 \leq t \leq T\}$ above;

Step 3. Construct bootstrap processes:

$$\tilde{y}_t = \begin{cases} \frac{1}{T\lambda}\sum\limits_{t=1}^{[T\lambda]} y_t + \tilde{\varepsilon}_t, & t \leq [m\lambda], \\ \frac{1}{T(1-\lambda)}\sum\limits_{t=[T\lambda]+1}^{T} y_t + \tilde{\varepsilon}_t, & t > [m\lambda]; \end{cases}$$

Step 4. Calculate the statistic $\xi_m(\lambda) = \frac{\frac{1}{m^2}\sum_{t=1}^{m}\{\sum_{s=1}^{t}\tilde{e}_s\}^2}{\frac{1}{m}\sum_{s=1}^{m}\tilde{e}_s^2}$, where $\{\tilde{e}_s, 1 \le s \le [m\lambda]\}$ are regression residuals of $\{\tilde{y}_s, 1 \le s \le [m\lambda]\}$ on a constant, $\{\tilde{e}_s, [m\lambda]+1 \le s \le m\}$ are residuals of $\{\tilde{y}_s, [m\lambda]+1 \le s \le m\}$ on a constant;

Step 5. Duplicate the steps above, we can calculate the empirical distribution and empirical p-values of $\xi_m(\lambda)$.

In order to prove the convergence of $\xi_m(\lambda)$, we adopt Athreya [10] assumption on the size of bootstrap procedures for an estimator in series with stable innovations:

Assumption When $T \to \infty$, $m \to \infty$ and $m/T \to \infty$.

3. Main results

Under the assumption above, four following results are established:

Lemma 1[11] *If $\{\varepsilon_t, t \ge 1\}$ and $\{\eta_t, t \ge 1\}$ are in an attracted field of a stable law, that is to say: there exists an $a_T = T^{1/\kappa} l(T)$, where $l(T)$ is a slowly varying function, such that*

$$a_T^{-1}\sum_{t=1}^{[Tr]}\varepsilon_t \to U_\kappa(r), a_T^{-1}\sum_{t=1}^{[Tr]}{\varepsilon_t}^2 \to V_{\kappa/2}(r), a_T^{-1}\sum_{t=1}^{[Tr]}\eta_t \to X_\kappa(r), a_T^{-1}\sum_{t=1}^{[Tr]}{\eta_t}^2 \to Y_{\kappa/2}(r),$$

where $U_\kappa(r), V_{\kappa/2}, X_\kappa(r), Y_{\kappa/2}(r)$ are stable variables with the corresponding index.

Theorem 1 *If $\{y_t\}$ are generated in model (1) under the hypothesis H_0, then the statistic satisfies:*

$$\xi_T(\lambda) \to \frac{\int_0^1 (B(r,\lambda))^2 \mathrm{d}r}{V_{\kappa/2}(1)},$$

where $U_\kappa(r), V_{\kappa/2}(1)$ are stable variables with the index $\kappa, \kappa/2$, respectively, and $B(r,\lambda)$ is defined as:

$$B(r,\lambda) = \begin{cases} U_\kappa(r) - \frac{r}{\lambda}U_\kappa(\lambda), & r \le \lambda, \\ \{U_\kappa(r) - U_\kappa(\lambda)\} - \frac{r-\lambda}{1-\lambda}\{U_\kappa(1) - U_\kappa(\lambda)\}, & r > \lambda. \end{cases}$$

Proof The residuals of $\{y_t\}$ generated by (1) on a constant is the equation (2): $e_t = \hat{\varepsilon}_t$, then by Lemma 1 and the theorem for continuous map of processes, $a_T^{-1}\sum_{t=1}^{[Tr]} e_t \to B(r,\lambda))$, and the numerator $T^{-1}a_T^{-2}\sum_{t=1}^{T}\{\sum_{s=1}^{t} e_s\}^2 \to \int_0^1 (B(r,\lambda))^2 \mathrm{d}r$ for

$$a_T^{-2}\sum_{s=1}^{[T\lambda]} e_t^2 = a_T^{-2}\sum_{s=1}^{[T\lambda]}\varepsilon_t^2 - (T\lambda)^{-1}(a_T^{-1}\sum_{s=1}^{[T\lambda]}\varepsilon_s)^2,$$

$$a_T^{-2}\sum_{s=[T\lambda]+1}^{T} e_t^2 = a_T^{-2}\sum_{s=[T\lambda]+1}^{T}\varepsilon_t^2 - (T(1-\lambda)^{-1}(a_T^{-1}\sum_{s=[T\lambda]+1}^{T}\varepsilon_s)^2.$$

The denominator $a_T^{-2}\sum_{s=1}^{T} e_s^2 \to V(1)$. The proof is completed. □

Theorem 2 *If $\{y_t\}$ are generated in model (1) under the alternative hypothesis H_1, then the statistic satisfies: $\xi_T(\lambda) = O_P(T)$.*

Proof Under the alternative hypothesis H_1, $\mu_k = \mu_{k-1} + \eta_t$ is a series of random walk, so the residuals of $\{y_t\}$ on a constant yields:

$$\hat{e}_t = \begin{cases} \mu_t - \frac{1}{T\lambda}\sum\limits_{t=1}^{[T\lambda]}\mu_t + \varepsilon_t - \frac{1}{T\lambda}\sum\limits_{t=1}^{[T\lambda]}\varepsilon_t, & t \le [T\lambda], \\ \mu_t - \frac{1}{T(1-\lambda)}\sum\limits_{t=[T\lambda]+1}^{T}\mu_t + \varepsilon_t - \frac{1}{T(1-\lambda)}\sum\limits_{t=[T\lambda]+1}^{T}\varepsilon_t, & t > [T\lambda]. \end{cases} \tag{2}$$

Then for $r \le \lambda$,

$$T^{-1}a_T^{-1}\sum_{s=1}^{[Tr]}(\mu_s - \frac{1}{T\lambda}\sum_{t=1}^{[T\lambda]}\mu_t) \to \int_0^r X_\kappa(s)\mathrm{d}s \frac{r}{\lambda}\int_\lambda^1 X_\kappa(s)\mathrm{d}s, \tag{3}$$

$$T^{-1}a_T^{-2}\sum_{s=1}^{[Tr]}(\mu_s - \frac{1}{T\lambda}\sum_{t=1}^{[T\lambda]}\mu_t)^2 \to \int_0^r X_\kappa^2(r)\mathrm{d}r - 2\lambda^{-1}\int_0^r X_\kappa(s)\mathrm{d}s\int_0^\lambda X_\kappa(s)\mathrm{d}s + \frac{r}{\lambda^2}(\int_0^\lambda X_\kappa(r)\mathrm{d}r)^2, \tag{4}$$

and for $r \ge \lambda$,

$$T^{-1}a_T^{-1}\sum_{s=[Tr]+1}^{T}(\mu_s - \frac{1}{T(1-\lambda)}\sum_{t=[T\lambda]+1}^{T}\mu_t) \to \int_r^1 X_\kappa(s)\mathrm{d}s\frac{r-\lambda}{1-\lambda}\int_\lambda^1 X_\kappa(s)\mathrm{d}s, \tag{5}$$

$$T^{-1}a_T^{-1}\sum_{s=[Tr]+1}^{T}(\mu_s - \frac{1}{T(1-\lambda)}\sum_{t=[T\lambda]+1}^{T}\mu_t)^2 \to \int_r^1 X_\kappa^2(r)\mathrm{d}r + \frac{1-r}{(1-\lambda)^2}(\int_\lambda^1 X_\kappa(r)\mathrm{d}r)^2 - 2(1-\lambda)^{-1}\int_r^1 X_\kappa(s)\mathrm{d}s\int_\lambda^1 X_\kappa(s)\mathrm{d}s. \tag{6}$$

From equations (3), (4) and (6),

$$T^{-1}a_T^{-1}\sum_{i=1}^{[Tr]}e_i \to B_1(r,\lambda), B_1(r,\lambda) = \begin{cases} \int_0^r X(s)\mathrm{d}s - \frac{r}{\lambda}\int_0^\lambda X_\kappa(s)\mathrm{d}s, & r \le \lambda, \\ \int_r^1 X(s)\mathrm{d}s - \frac{r-\lambda}{1-\lambda}\int_\lambda^1 X_\kappa(s)\mathrm{d}s, & r > \lambda. \end{cases} \tag{7}$$

So $T^{-2}a_T^{-2}\sum_{t=1}^T\{\sum_{s=1}^t e_s\}^2 = O_P(Ta_T^2)$. From equations (3), (5) and (7), $T^{-1}a_T^{-2}\sum_{s=1}^T e_s^2 = O_P(1)$. The proof of Theorem 2 is completed. □

Remark 1 Theorem 1 is just the asymptotical distribution in Busetti [8] when the index $\kappa = 2$. Theorem 2 states the consistency of the test.

For convenience, we denote by $\xi_\infty(\lambda)$ the asymptotical distribution of $\xi_T(\lambda)$. Let $\varepsilon = \sigma(\varepsilon_t,\ t \ge 1)$ and P_ε be a conditional probability on ε. Under the hypothesis H_0,

$$\hat{\varepsilon}_t = \begin{cases} \varepsilon_t - \frac{1}{T\lambda}\sum\limits_{t=1}^{[T\lambda]}\varepsilon_t, & t \le [T\lambda], \\ \varepsilon_t - \frac{1}{T(1-\lambda)}\sum\limits_{t=[T\lambda]+1}^{T}\varepsilon_t, & t > [T\lambda], \end{cases} \tag{8}$$

so the corresponding unobservable variable ε_t is selected when an $\tilde{\varepsilon}_t$ is selected, denoted by $\underline{\varepsilon}_t$. The following lemma is necessary for convergence of bootstrap procedures:

Lemma 2 *Under the assumption and hypothesis H_0, if $U_m(\tau) = a_m^{-1}\sum_{i=1}^{[m\tau]}\tilde{\varepsilon}_i$, $0 \le \tau \le 1$, for* *any bounded continuous function h on $D[0,1]$, $P_\varepsilon(h(a_m^{-1}\sum_{i=1}^{[m\tau]}\tilde{\varepsilon}_i)\le x)\to P(h(U(\cdot))\le x)$ for all points of continuity of the stable law x.*

Proof From the definition of $\{\tilde{\varepsilon}_t\}$ and the equation (2),

$$\tilde{\varepsilon}_t=\begin{cases}\underline{\varepsilon}_t-\frac{1}{T\lambda}\sum\limits_{t=1}^{[T\lambda]}\varepsilon_t, & t\le[m\lambda],\\ \underline{\varepsilon}_t-\frac{1}{T(1-\lambda)}\sum\limits_{t=[T\lambda]+1}^{T}\varepsilon_t, & t>[m\lambda],\end{cases}\tag{9}$$

so $a_m^{-1}\sum_{i=1}^{[m\tau]}\tilde{\varepsilon}_i=a_m^{-1}\sum_{i=1}^{[m\tau]}\underline{\varepsilon}_i-\frac{[m\tau]}{T\lambda}\sum_{i=1}^{[T\lambda]}\varepsilon_i-\frac{[m(\tau-\lambda]}{T(1-\lambda)}\sum_{[T\lambda]+1}^{T}\varepsilon_t$. Lemma 1 and the assumption certify that the last two terms converge to 0 in probability. With the Lemma 1 in [10], the proof is completed. □

Theorem 3 *Under the assumption and the hypothesis H_0, for any $x>0$, the empirical distribution satisfies: $P_\varepsilon(\xi_m(\lambda)\le x)\to P_\varepsilon(\xi_\infty(\lambda)\le x)$, $\quad T\to\infty$.*

Proof For $\xi_m(\lambda)=\frac{\frac{1}{m^2}\sum_{t=1}^{m}\{\sum_{s=1}^{t}\tilde{e}_s\}^2}{\frac{1}{m}\sum_{s=1}^{m}\tilde{e}_s^2}$, just like the way the numerator is considered, the denominator can be considered similarly. Under the hypothesis H_0,

$$\tilde{\varepsilon}_t=\begin{cases}\tilde{y}_t-\frac{1}{m\lambda}\sum\limits_{t=1}^{[m\lambda]}\tilde{y}_t=\tilde{\varepsilon}_t-\frac{1}{m\lambda}\sum\limits_{t=1}^{[m\lambda]}\tilde{\varepsilon}_t, & t\le[m\lambda],\\ \tilde{y}_t-\frac{1}{m(1-\lambda)}\sum\limits_{t=[m\lambda]+1}^{m}\tilde{y}_t=\tilde{\varepsilon}_t-\frac{1}{m(1-\lambda)}\sum\limits_{t=[m\lambda]+1}^{m}\tilde{\varepsilon}_t, & t>[m\lambda],\end{cases}$$

with Lemma 2 and the theorem for continuous map of processes, $a_m^{-1}\sum_{s=1}^{[mr]}\tilde{e}_s\to B(r,\lambda)$, then

$$ma_m^{-2}\{\frac{1}{m^2}\sum_{t=1}^{m}(\sum_{s=1}^{t}\tilde{e}_s)^2\}\to\int_0^1(B(r,\lambda))^2.$$

The proof is completed. □

Theorem 4 *Under the assumption and the alternative hypothesis H_1, the empirical distribution satisfies: $\xi_m(\lambda)=O_P(m)$.*

Proof Under the alternative hypothesis H_1, denote the corresponding μ_t as $\tilde{\mu}_t$ when the bootstrap sample $\tilde{\varepsilon}_t$ is selected. With the definition of $\tilde{\varepsilon}_t$ and equation (3),

$$\tilde{\varepsilon}_s=\begin{cases}\tilde{\mu}_s-\frac{1}{T\lambda}\sum\limits_{t=1}^{[T\lambda]}\mu_t+\underline{\varepsilon}_t-\frac{1}{T\lambda}\sum\limits_{t=1}^{[T\lambda]}\varepsilon_t, & s\le[T\lambda],\\ \tilde{\mu}_s-\frac{1}{T(1-\lambda)}\sum\limits_{t=[T\lambda]+1}^{T}\mu_t+\underline{\varepsilon}_t-\frac{1}{T(1-\lambda)}\sum\limits_{t=[T\lambda]+1}^{T}\varepsilon_t, & t>[T\lambda],\end{cases}\tag{10}$$

then

$$\tilde{e}_s=\begin{cases}\tilde{y}_s-\frac{1}{m\lambda}\sum\limits_{t=1}^{[m\lambda]}\tilde{y}_t=\tilde{\mu}_s-\frac{1}{m\lambda}\sum\limits_{t=1}^{[m\lambda]}\tilde{\mu}_t+\underline{\varepsilon}_s-\frac{1}{m\lambda}\sum\limits_{t=1}^{[m\lambda]}\tilde{\varepsilon}_t, \quad t\le[m\lambda],\\ \tilde{y}_s-\frac{1}{m(1-\lambda)}\sum\limits_{t=[m\lambda]+1}^{m}\tilde{y}_t=\tilde{\mu}_s-\frac{1}{m(1-\lambda)}\sum\limits_{t=[m\lambda]+1}^{m}\tilde{\mu}_t+\underline{\varepsilon}_s-\\ \qquad\frac{1}{m(1-\lambda)}\sum\limits_{t=[m\lambda]+1}^{m}\tilde{\varepsilon}_t, \quad t>[m\lambda].\end{cases}\tag{11}$$

Following the similar procedures in Theorem 3.2, we get

$$m^{-2}a_m^{-2}\sum_{t=1}^{m}\{\sum_{s=1}^{t}\tilde{e}_s\}^2 = O_P(ma_m^2), \quad m^{-1}a_m^{-2}\sum_{s=1}^{m}e_s^2 = O_P(1).$$

The proof is completed. □

Remark 2 Theorem 3 points out that the empirical bootstrap distribution is a nice approximation of $\xi_\infty(\lambda)$. Theorem 4 states that the power of bootstrap procedures has no loss asymptotically.

4. Simulations

In this section, we study the performance of the test and corresponding Bootstrap procedures through simulations for stable innovations with the index $\kappa = 1.14$. Critic values in Table 1 are obtained by simulating the asymptotical distribution directly, namely, we calculate the statistic 5000 times independently with a sample size 1000 under the null hypothesis H_0, then use the empirical distribution of the statistic to obtain the values in Table 1. From Table 1, critic values for $\kappa = 1.14$ are larger than those for $\kappa = 2$, so the heaviness of the innovation will affect the statistic in Busetti [8]. In Table 2 and 3, we employ two functions, $m = [T/LnT]$ and $m = [T/Ln(LnT)]$ to decide the size of bootstrap sample. From Table 2 and 3, critic values by Bootstrap procedures are nearly equal to the critic values by simulating directly, just as Theorem 3. Besides this, critic values by Bootstrap procedures are affected less than those by simulating directly by the ourliers in the innovations. In Table 4, we compare the power of the simulation directly with the Bootstrap procedures. From Table 4, the empirical power of the Bootstrap procedures is higher than that of simulation directly asymptotically, as stated in Theorem 4.

Appendix

	1%	2.5%	5%	10%	90%	95%	97.5%	99%
λ=0.1	0.0292	0.0372	0.0464	0.0592	0.3556	0.4579	0.5724	0.7242
0.2	0.0303	0.0403	0.0503	0.0645	0.4757	0.6448	0.8056	1.0024
0.3	0.0316	0.0391	0.0488	0.0639	0.5775	0.7645	0.9441	1.1424
0.4	0.0273	0.0380	0.0476	0.0637	0.6565	0.8734	1.0659	1.3566
0.5	0.0336	0.0401	0.0485	0.0631	0.6800	0.9188	1.1790	1.4841
0.6	0.0321	0.0409	0.0506	0.0658	0.6600	0.9135	1.1475	1.4668
0.7	0.0312	0.0393	0.0500	0.0653	0.5946	0.7911	0.9928	1.2605
0.8	0.0328	0.0404	0.0502	0.0633	0.4751	0.6285	0.7929	1.0345
0.9	0.0283	0.0382	0.0468	0.0591	0.3655	0.4677	0.5819	0.7460

Table 1 Critic values by Monte Carlo simulation, T=1000, $\kappa = 1.14$

	1%	2.5%	5%	10%	90%	95%	97.5%	99%
λ=0.1	0.0271	0.0339	0.0407	0.0514	0.3505	0.4538	0.5435	0.6579
0.2	0.0283	0.0345	0.0416	0.0512	0.3477	0.4416	0.5386	0.6886
0.3	0.0319	0.0404	0.0499	0.0630	0.5120	0.6621	0.7900	0.9816
0.4	0.0293	0.0361	0.0446	0.0570	0.5041	0.6641	0.8122	1.0284
0.5	0.0364	0.0457	0.0565	0.0764	0.8316	1.0495	1.2718	1.5214
0.6	0.0292	0.0355	0.0436	0.0579	0.4848	0.6318	0.7803	0.9477
0.7	0.0299	0.0379	0.0459	0.0594	0.4967	0.6700	0.8240	1.0327
0.8	0.0315	0.0395	0.0489	0.0615	0.4869	0.6202	0.7765	0.9762
0.9	0.0311	0.0385	0.0462	0.0567	0.3701	0.4630	0.5914	0.7365

Table 2 Critic values by Bootstrap procedure, T=1000, $m = [T/LnT] = 144$, $\kappa = 1.14$

	1%	2.5%	5%	10%	90%	95%	97.5%	99%
λ=0.1	0.0295	0.0350	0.0423	0.0529	0.3660	0.4751	0.5775	0.6968
0.2	0.0465	0.0609	0.0786	0.1106	0.7827	0.9194	1.0484	1.2410
0.3	0.0545	0.0737	0.1000	0.1378	0.8717	1.0622	1.2394	1.5063
0.4	0.0278	0.0351	0.0456	0.0611	0.5239	0.6846	0.8595	1.1140
0.5	0.0328	0.0404	0.0474	0.0586	0.9071	1.1595	1.3658	1.6451
0.6	0.0276	0.0337	0.0400	0.0503	0.4552	0.6393	0.8082	0.9993
0.7	0.0330	0.0411	0.0520	0.0683	0.4693	0.6249	0.7638	1.0315
0.8	0.0524	0.0659	0.0792	0.1037	0.7409	0.9286	1.1149	1.3145
0.9	0.0294	0.0349	0.0416	0.0506	0.3513	0.4611	0.5669	0.6881

Table 3 Critic values by Bootstrap procedure, T=1000, $m = [T/LnLnT] = 517$, $\kappa = 1.14$

	Monte Carlo				bootstrap			
	10%	5%	2.5%	1%	10%	5%	2.5%	1%
λ=0.1	0.9980	0.9976	0.9974	0.9962	0.9992	0.9992	0.9988	0.9978
0.2	0.9984	0.9976	0.9970	0.9962	0.9986	0.9984	0.9976	0.9960
0.3	0.9964	0.9950	0.9938	0.9930	0.9986	0.9978	0.9972	0.9966
0.4	0.9970	0.9962	0.9954	0.9948	0.9962	0.9952	0.9944	0.9934
0.5	0.9970	0.9962	0.9950	0.9944	0.9962	0.9952	0.9936	0.9926
0.6	0.9972	0.9962	0.9956	0.9946	0.9972	0.9964	0.9956	0.9952
0.7	0.9972	0.9966	0.9956	0.9944	0.9974	0.9970	0.9964	0.9954
0.8	0.9976	0.9970	0.9964	0.9952	0.9982	0.9978	0.9976	0.9968
0.9	0.9992	0.9988	0.9986	0.9982	0.9982	0.9980	0.9968	0.9960

Table 4 Empirical power of two methods ($\kappa = 1.14, m = 144$)

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