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BOUNDEDNESS OF TOEPLITZ OPERATORS GENERATED BY THE CAMPANATO-TYPE FUNCTIONS AND RIESZ TRANSFORMS ASSOCIATED WITH SCHÖDINGER OPERATORS

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Abstract: In the paper, we study the boundedness of the Toeplitz operators generated by the Campanato-type functions and Riesz transforms associated with the Schrödinge operators. Using the sharp maximal function estimate, we establish the boundedness of the Toeplitz operator Θ^b on the Lebesgue space, which extend the previous results about the comutators.

Keywords: Commutator; Campanato-type functions; Riesz transform; Schrödinger operator

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1 Introduction

Let $\mathfrak{L} = -\triangle + V$ be a Schrödinger operator on $\mathbb{R}^n (n > 3)$, where $\triangle$ is the Laplacian on $\mathbb{R}^n$ and $V \neq 0$ is a nonnegative locally integrable function. The problems about the Schrödinger operators $\mathfrak{L}$ were well studied (see [1–3] for example). Especially, Fefferman [1], Shen [2] and Zhong [3] developed some basic results.

The commutators generated by the Riesz transforms associated with the Schrödinger operators and BMO functions or Lipschitz functions also attracted much attention (see [4–6] for example). Chu [7], consider the boundedness of commutators generalized by the $\mathrm{BMO}_{\mathfrak{L}}$ functions and the Riesz transform $\nabla(-\triangle + V)^{-1/2}$ on Lebesgue spaces. Mo et al. [8] established the boundedness of commutators generated by the Campanato-type functions and the Riesz transforms associated with Schrödinger operators.

First, let us introduce some notations. A nonnegative locally $L^q(\mathbb{R}^n)$ integrable function V is said to belong to $B_q (1 < q < \infty)$ if there exists $C = C(q, V) > 0$ such that the reverse Hölder's inequality

$$\left(\frac{1}{|B|}\int_B V(x)^q dx\right)^{1/q} \leq C\left(\frac{1}{|B|}\int_B V(x)dx\right) \tag{1.1}$$

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holds for every ball B in $\mathbb{R}^n$.

Let $T_{j,1} = \nabla(-\triangle + V)^{-1/2}$ or $T_{j,1} = \pm I$ and $\mathrm{T}_{j,2}$ be a linear operator which is bounded on $\mathrm{L}^p(\mathbb{R}^n)$ space, for $j = 1, 2, \cdots, m$. Let $M_b f = bf$, where b is a locally integrable function on $\mathbb{R}^n$, then the Toeplitz operator is defined by

$$\Theta^b = \sum_{j=1}^{m} T_{j,1} M_b T_{j,2}.$$

About the Toeplitz operator, there are some results. Mo et al. [9] established the boundedness of the Toeplitz operator generalized by the singular integral operator with nonsmooth kernel and the generalized fractional. Liu et al. [11] investigated the boundness of the Toeplitz operator related to the generalized fractional integral operator.

The commutator $[b, T](f) = bT(f) - T(bf)$ is a particular case of the Toeplitz operators. Inspired by [7–10], we will consider the boundedness of the Toeplitz operators generated by the Campanato-type functions and Riesz transforms associated with Schödinger operators.

Definition 1.1 Let $f \in L_{\mathrm{loc}}(\mathbb{R}^n)$, then the sharp maximal function associated with $\mathfrak{L} = -\triangle + V$ is defined by

$$M_{\mathfrak{L}}^{\#}(f)(x) = \begin{cases} \sup\limits_{x \in B} \dfrac{1}{|B|} \displaystyle\int_{B(x,s)} |f(y) - f_{B(x,s)}| dy, & \text{when } s < \rho(x), \\ \sup\limits_{x \in B} \dfrac{1}{|B(x,s)|} \displaystyle\int_{B(x,s)} |f(y)| dy, & \text{when } s \geq \rho(x), \end{cases}$$

where ρ is define by

$$\rho(x) = \sup\left\{ r > 0 : \frac{1}{r^{n-2}} \int_{B(x,r)} V \leq 1 \right\}.$$

Definition 1.2 [12, 13] Let $\mathfrak{L} = -\triangle + V$, $p \in (0, \infty)$ and $\beta \in \mathbb{R}$. A function $f \in L^p_{\mathrm{loc}}(\mathbb{R}^n)$ is said to be in $\Lambda_{\mathfrak{L}}^{\beta,p}(\mathbb{R}^n)$, if there exists a nonnegative constant C such that for all $x \in \mathbb{R}^n$ and $0 < s < \rho(x) \leq r$,

$$\left\{ \frac{1}{|B(x,s)|^{1+p\beta}} \int_{B(x,s)} |f(y) - f_{B(x,s)}|^p dy \right\}^{1/p} + \left\{ \frac{1}{|B(x,r)|^{1+p\beta}} \int_{B(x,r)} |f(y)|^p dy \right\}^{1/p} \leq C,$$

where $f_B = \frac{1}{|B|} \int_B f(y) dy$ for any ball B. Moreover, the minimal constant C as above is defined for the norm of f in the space $\Lambda_{\mathfrak{L}}^{\beta,p}(\mathbb{R}^n)$ and denote by $\|f\|_{\Lambda_{\mathfrak{L}}^{\beta,p}(\mathbb{R}^n)}$.

Remak 1.1 [12] When $p \in [1, \infty)$, $\Lambda_{\mathfrak{L}}^{0,p}(\mathbb{R}^n) = \mathrm{BMO}_{\mathfrak{L}}(\mathbb{R}^n)$. And, when $0 \leq \beta < \infty$ and $p_1, p_2 \in [1, \infty)$, $\Lambda_{\mathfrak{L}}^{\beta,p_1}(\mathbb{R}^n) = \Lambda_{\mathfrak{L}}^{\beta,p_2}(\mathbb{R}^n)$ and $\|f\|_{\Lambda_{\mathfrak{L}}^{\beta,p_1}(\mathbb{R}^n)} \sim \|f\|_{\Lambda_{\mathfrak{L}}^{\beta,p_2}(\mathbb{R}^n)}$. For simplicity, we denote $\Lambda_{\mathfrak{L}}^{\beta,p}(\mathbb{R}^n)$ by $\Lambda_{\mathfrak{L}}^{\beta}(\mathbb{R}^n)$.

Lemma 1.1 (see [2, 7]) Suppose that $V \in B_q(n/2 \leq q < n)$ satisfies the condition

$$\int_{B(x,R)} \frac{V(y)}{|x-y|^{n-1}} dy \leq \frac{1}{R^{n-1}} \int_{B(x,R)} V(y) dy, \tag{1.2}$$

then the kernel $K(x,y)$ of operator $\nabla(-\triangle+V)^{-1/2}$ satisfies the following estimates: there exists a constant $\delta>0$ such that for any nonnegative integrate i,

$$|K(x,y)|\le \frac{C_i}{\{|1+m(x,V)|x-y|\}^i}\frac{1}{|x-y|^n}, \tag{1.3}$$

$$|K(x+h,y)-K(x,y)|\le C\frac{|h|^\delta}{|x-y|^{n+\delta}} \tag{1.4}$$

for $0<|h|<\frac{|x-y|}{2}$.

Hence, $\nabla(-\triangle+V)^{-1/2}$ is boundedness on $L^p(\mathbb{R}^n)$ space for $1<p\le p_0$, where $1/p_0=1/q-1/n$.

Throughout this paper, the letter C always remains to denote a positive constant that may vary at each occurrence but is independent of the essential variable.

2 Theorems and Lemmas

Theorems 2.1 Let $V\in B_q$ satisfy (1.2) for $n/2\le q<n$. Let $0\le\beta<1$, $b\in\Lambda_{\mathfrak{L}}^\beta(\mathbb{R}^n)$, $1<\tau<\infty$ and $1<s<p_0$, where $1/p_0=1/q-1/n$. Suppose that $\Theta^1 f=0$ for any $f\in L^r(\mathbb{R}^n)$ $(1<r<\infty)$, then there exists a constant $C>0$ such that

$$M_{\mathfrak{L}}^{\#}(\Theta^b f)(x)\le C\sum_{j=1}^m \|b\|_{\Lambda_{\mathfrak{L}}^\beta} M_{s\tau,n\beta}(T_{j,2}f)(x).$$

Theorems 2.2 Let $V\in B_q$ satisfy (1.2) for $n/2\le q<n$ and $1<p_0<\infty$ satisfy $1/p_0=1/q-1/n$. Suppose that $\Theta^1 f=0$ for any $f\in L^\tau(\mathbb{R}^n)$ $(1<\tau<\infty)$, $0<\beta<1$, and $b\in\Lambda_{\mathfrak{L}}^\beta(\mathbb{R}^n)$. Then for $1<r<\min\{1/\beta,p_0\}$ and $1/p=1/r-\beta$, there exists a constant $C>0$, such that

$$\|\Theta^b f\|_{L^p}\le C\sum_{j=1}^m \|b\|_{\Lambda_{\mathfrak{L}}^\beta}\|f\|_{L^r}.$$

Theorems 2.3 Let $V\in B_q$ satisfy (1.2) for $n/2\le q<n$ and $1<p_0<\infty$ satisfy $1/p_0=1/q-1/n$. Suppose that $\Theta^1 f=0$ for any $f\in L^\tau(\mathbb{R}^n)$ $(1<\tau<\infty)$ and $b\in BMO_{\mathfrak{L}}(\mathbb{R}^n)$, then for $1<p<p_0$, there exists a constant $C>0$, such that

$$\|\Theta^b f\|_{L^p}\le C\sum_{j=1}^m \|b\|_{\mathrm{BMO}_{\mathfrak{L}}}\|f\|_{L^p}.$$

To prove the theorems, we need the following lemmas.

Lemma 2.1 (see [7]) Let $0<p_0<\infty$, $p_0\le p<\infty$ and $\delta>0$. If f satisfies the condition $M(|f|^\delta)^{1/\delta}\in L^{p_0}$, then exists a constant $C>0$ such that

$$\|M(|f|^\delta)^{1/\delta}\|_{L^p}\le C\|M_{\mathfrak{L}}^{\#}(|f|^\delta)^{1/\delta}\|_{L^p}.$$

Lemma 2.2 (see [14]) For $1\le\gamma<\infty$ and $\beta>0$, let

$$M_{\gamma,\beta}(f)(x)=\sup_{B\ni x}\left(\frac{1}{|B|^{1-\beta\gamma/n}}\int_B |f(y)|^\gamma dy\right)^{1/\gamma}.$$

Suppose that $\gamma < p < n/\beta$ and $1/q = 1/p - \beta/n$, then $\|M_{\gamma,\beta}(f)\|_{L^q} \le C\|f\|_{L^p}$.

Remark 2.1 When $\beta = 0$, we denote $M_{\gamma,\beta} = M_r$. And it is easy to see that M_r is boundedness on $L^p(\mathbb{R}^n)$, for $1 < r < p$.

Lemma 2.3 (see [8]) Let $B = B(x,r)$ and $0 < r < \rho(x)$, then

$$|b_{2^kB} - b_B| \le Ck|2^kB|^{\beta}\|b\|_{\Lambda^{\beta}_{\mathfrak{L}}} \text{ for } k = 1, 2, \cdots .$$

3 Proofs of Theorems 2.1–2.2

First, let us prove Theorem 2.1.

Fix a ball $B = B(x, r_0)$ and let $2B = B(x, 2r_0)$. We need only to estimate

$$\frac{1}{|B|}\int_B |\Theta^b f(y) - (\Theta^b f)_B|dy.$$

Case I When $0 < r_0 < \rho(x)$, using the condition $\Theta^1 f = 0$, then we have

$$\begin{aligned}
&\frac{1}{|B|}\int_B |\Theta^b f(y) - (\Theta^b f)_B|dy\\
\le\ &\sum_{j=1}^m \frac{1}{|B|}\left(\int_B |T_{j,1}M_{(b-b_B)}T_{j,2}f(y) - (T_{j,1}M_{(b-b_B)}T_{j,2}f)_B|dy\right).
\end{aligned}$$

If $T_{j,1} = \nabla(-\triangle + V)^{-1/2}$, then

$$\begin{aligned}
&\frac{1}{|B|}\int_B |T_{j,1}M_{(b-b_B)}T_{j,2}f(y) - (T_{j,1}M_{(b-b_B)}T_{j,2}f)_B|dy\\
\le\ &\frac{2}{|B|}\int_B |T_{j,1}M_{(b-b_B)}T_{j,2}f(y) - T_{j,1}M_{(b-b_B)\chi_{R^n\setminus 2B}}T_{j,2}f(x)|dy\\
\le\ &\frac{2}{|B|}\int_B |T_{j,1}M_{(b-b_B)\chi_{2B}}T_{j,2}f(y)|dy + \frac{2}{|B|}\int_B |T_{j,1}M_{(b-b_B)\chi_{R^n\setminus 2B}}T_{j,2}f(y)\\
&-T_{j,1}M_{(b-b_B)\chi_{R^n\setminus 2B}}T_{j,2}f(x)|dy\\
=:\ &I_1 + I_2.
\end{aligned}$$

Let τ and s be as in Theorem 2.1. Then using Hölder's inequality and the L^s boundedness of $T_{j,1}$ (Lemma 1.1), we have

$$\begin{aligned}
I_1 &\le C\left(\frac{1}{|B|}\int_B |T_{j,1}M_{(b-b_B)\chi_{2B}}T_{j,2}f(y)|^s dy\right)^{\frac{1}{s}}\\
&\le C\left(\frac{1}{|2B|^{1+\beta s\tau'}}\int_{2B} |b(y) - b_B|^{s\tau'} dy\right)^{\frac{1}{s\tau'}}\left(\frac{1}{|2B|^{1-\beta s\tau}}\int_{2B} |T_{j,2}f(y)|^{s\tau} dy\right)^{\frac{1}{s\tau}}\\
&\le C\|b\|_{\Lambda^{\beta}_{\mathfrak{L}}} M_{s\tau,n\beta}(T_{j,2}f)(x).
\end{aligned}$$

Let's estimate I_2. From (1.4), it follows that

$$\begin{aligned}
& |T_{j,1}[(b-b_B)\chi_{R^n\setminus 2B}T_{j,2}f](y) - T_{j,1}[(b-b_B)\chi_{R^n\setminus 2B}T_{j,2}f](x)| \\
= & \left|\int_{(2B)^c}(b(z)-b_B)T_{j,2}f(z)(K(y,z)-K(x,z))dz\right| \\
\leq & C\sum_{k=1}^{\infty}\int_{2^k r_0<|z-x|\leq 2^{k+1}r_0}\frac{|y-x|^\delta}{|z-x|^{n+\delta}}|b(z)-b_B||T_{j,2}f(z)|dz \\
\leq & C\sum_{k=1}^{\infty}\frac{r_0^\delta}{(2^k r_0)^{n+\delta}}\int_{|z-x|\leq 2^{k+1}r_0}|b(z)-b_{2^{k+1}B}||T_{j,2}f(z)|dz \\
& +C\sum_{k=1}^{\infty}\frac{r_0^\delta}{(2^k r_0)^{n+\delta}}\int_{|z-x|\leq 2^{k+1}r_0}|b_{2^{k+1}B}-b_B||T_{j,2}f(z)|dz \\
=: & H_1+H_2.
\end{aligned}$$

For H_1, since $\delta>0$, by Hölder's inequality, we have

$$\begin{aligned}
H_1 & \leq C\sum_{k=1}^{\infty}2^{-k\delta}\left(\frac{1}{|2^{k+1}B|^{1+\beta(s\tau)'}}\int_{2^{k+1}B}|b(y)-b_{2^{k+1}B}|^{(s\tau)'}dy\right)^{1/(s\tau)'} \\
& \quad\times\left(\frac{1}{|2^{k+1}B|^{1-\beta s\tau}}\int_{2^{k+1}B}|T_{j,2}f(y)|^{s\tau}dy\right)^{1/s\tau} \\
& \leq C\|b\|_{\Lambda_{\mathfrak{L}}^{\beta}}M_{s\tau,n\beta}(T_{j,2}f)(x).
\end{aligned}$$

From Lemma 2.3 and Hölder's inequality, it follows that

$$\begin{aligned}
H_2 & \leq C\|b\|_{\Lambda_{\mathfrak{L}}^{\beta}}\sum_{k=1}^{\infty}k2^{-k\delta}\frac{1}{|2^{k+1}B|^{1-\beta}}\int_{2^{k+1}B}|T_{j,2}f(y)|dy \\
& \leq C\|b\|_{\Lambda_{\mathfrak{L}}^{\beta}}\sum_{k=1}^{\infty}k2^{-k\delta}\left(\frac{1}{|2^{k+1}B|^{1-\beta s\tau}}\int_{2^{k+1}B}|T_{j,2}f(y)|^{s\tau}dy\right)^{\frac{1}{s\tau}} \\
& \leq C\|b\|_{\Lambda_{\mathfrak{L}}^{\beta}}M_{s\tau,n\beta}(T_{j,2}f)(x).
\end{aligned}$$

Thus

$$I_2\leq C\|b\|_{\Lambda_{\mathfrak{L}}^{\beta}}M_{s\tau,n\beta}(T_{j,2}f)(x).$$

So when $T_{j,1}=\nabla(-\triangle+V)^{-1/2}$, we conclude that

$$\frac{1}{|B|}\int_B|T_{j,1}M_{(b-b_B)}T_{j,2}f(y)-(T_{j,1}M_{(b-b_B)}T_{j,2}f)_B|dy\leq C\|b\|_{\Lambda_{\mathfrak{L}}^{\beta}}M_{s\tau,n\beta}(T_{j,2}f)(x).$$

If $T_{j,1}=\pm I$, it is obvious that

$$\frac{2}{|B|}\int_B|T_{j,1}M_{(b-b_B)\chi_{R^n\setminus 2B}}T_{j,2}f(y)|dy=0.$$

Thus using the above formula and Hölder's inequality, we conclude

$$\begin{aligned} & \frac{1}{|B|}\int_B |T_{j,1}M_{(b-b_B)}T_{j,2}f(y)-(T_{j,1}M_{(b-b_B)}T_{j,2}f)_B|dy \\ \leq\ & \frac{2}{|B|}\int_B |T_{j,1}M_{(b-b_B)\chi_{2B}}T_{j,2}f(y)|dy \\ \leq\ & C\bigg(\frac{1}{|B|^{1+\beta(s\tau)'}}\int_B |b(y)-b_B|^{(s\tau)'}dy\bigg)^{\frac{1}{(s\tau)'}}\bigg(\frac{1}{|B|^{1-\beta s\tau}}\int_B |T_{j,2}f(y)|^{s\tau}dy\bigg)^{\frac{1}{s\tau}} \\ \leq\ & C\|b\|_{\Lambda_{\mathfrak{L}}^{\beta}}M_{s\tau,n\beta}(T_{j,2}f)(x). \end{aligned}$$

Thus for $0<r_0<\rho(x)$, we conclude that

$$M_{\mathfrak{L}}^{\#}(\Theta^b f)(x)\leq C\sum_{j=1}^{m}\|b\|_{\Lambda_{\mathfrak{L}}^{\beta}}M_{s\tau,n\beta}(T_{j,2}f)(x).$$

Case II When $r_0>\rho(x)$, we have

$$\frac{1}{|B|}\int_B |\Theta^b f(y)|dy\leq\sum_{j=1}^{m}\bigg(\frac{1}{|B|}\int_B |T_{j,1}M_{b\chi_{2B}}T_{j,2}f(y)|dy+\frac{1}{|B|}\int_B |T_{j,1}M_{b\chi_{R^n\backslash 2B}}T_{j,2}f(y)|dy\bigg).$$

If $T_{j,1}=\nabla(-\triangle+V)^{-1/2}$, then for $1<\tau<\infty$ and $1<s<p_0$ are as in Theorem 2.1, we have

$$\begin{aligned} & \frac{1}{|B|}\int_B |T_{j,1}M_{b\chi_{2B}}T_{j,2}f(y)|dy \\ \leq\ & C\bigg(\frac{1}{|B|}\int_B |T_{j,1}b\chi_{2B}T_{j,2}f(y)|^s dy\bigg)^{\frac{1}{s}} \\ \leq\ & \bigg(\frac{1}{|B|}\int_{2B}|b(y)|^s|T_{j,2}f(y)|^s dy\bigg)^{\frac{1}{s}} \\ \leq\ & C\bigg(\frac{1}{|2B|^{1+\beta s\tau'}}\int_{2B}|b(y)|^{s\tau'}dy\bigg)^{\frac{1}{s\tau'}}\bigg(\frac{1}{|2B|^{1-\beta s\tau}}\int_{2B}|T_{j,2}f(y)|^{s\tau}dy\bigg)^{\frac{1}{s\tau}} \\ \leq\ & C\|b\|_{\Lambda_{\mathfrak{L}}^{\beta}}M_{s\tau,n\beta}(T_{j,2}f)(x). \end{aligned}$$

Since when $y\in 2B$ and $z\in 2^{k+1}B\backslash 2^kB$, we have $|x-z|\sim|y-z|$. Then taking s,τ as above, we get

$$\begin{aligned} & |T_{j,1}M_{b\chi_{R^n\backslash 2B}}T_{j,2}f(y)|\leq\int_{(2B)^c}|b(z)||T_{j,2}f(z)||K(y,z)|dz \\ \leq\ & C\sum_{k=1}^{\infty}\int_{2^kr_0<|z-x|\leq 2^{k+1}r_0}\frac{C_i}{\{1+|z-x|m(x,V)\}^i|z-x|^n}|b(z)||T_{j,2}f(z)|dz \\ \leq\ & C\sum_{k=1}^{\infty}\frac{1}{2^{ki}}\bigg(\frac{1}{|2^{k+1}B|^{1+\beta(s\tau)'}}\int_{2^{k+1}B}|b(y)|^{(s\tau)'}dy\bigg)^{1/(s\tau)'} \\ & \bigg(\frac{1}{|2^{k+1}B|^{1-\beta s\tau}}\int_{2^{k+1}B}|T_{j,2}f(y)|^{s\tau}dy\bigg)^{1/s\tau} \\ \leq\ & C\|b\|_{\Lambda_{\mathfrak{L}}^{\beta}}M_{s\tau,n\beta}(T_{j,2}f)(x). \end{aligned}$$

Thus

$$\frac{1}{|B|}\int_B |T_{j,1}M_{b\chi_{R^n\setminus 2B}}T_{j,2}f(y)|dy \le C\|b\|_{\Lambda^\beta_{\mathfrak{L}}} M_{s\tau,n\beta}(T_{j,2}f)(x).$$

If $T_{j,1}=\pm I$, then by Hölder's inequality, we obtain

$$\begin{aligned} &\frac{1}{|B|}\int_B |T_{j,1}M_{b\chi_{2B}}T_{j,2}f(y)|dy \\ \le\ & C\Big(\frac{1}{|B|^{1+\beta(s\tau)'}}\int_B |b(y)|^{(s\tau)'}dy\Big)^{\frac{1}{(s\tau)'}}\Big(\frac{1}{|B|^{1-\beta s\tau}}\int_B |T_{j,2}f(y)|^{s\tau}dy\Big)^{\frac{1}{s\tau}} \\ \le\ & C\|b\|_{\Lambda^\beta_{\mathfrak{L}}} M_{s\tau,n\beta}(T_{j,2}f)(x). \end{aligned}$$

And

$$\frac{1}{|B|}\int_B |T_{j,1}M_{b\chi_{R^n\setminus 2B}}T_{j,2}(f)(y)|dy = 0.$$

Thus for $r_0>\rho(x)$,

$$M^{\#}_{\mathfrak{L}}(\Theta^b f)(x) \le C\sum_{j=1}^m \|b\|_{\Lambda^\beta_{\mathfrak{L}}} M_{s\tau,n\beta}(T_{j,2}f)(x).$$

So whenever $0<r_0<\rho(x)$ or $r_0>\rho(x)$, we have

$$M^{\#}_{\mathfrak{L}}(\Theta^b f)(x) \le C\sum_{j=1}^m \|b\|_{\Lambda^\beta_{\mathfrak{L}}} M_{s\tau,n\beta}(T_{j,2}f)(x).$$

Now, let us turn to prove Theorem 2.2.

Let s,τ be as in Theorem 2.1 and satisfy $1<s\tau<p$. Then applying Theorem 2.1, Lemma 2.1 and Lemma 2.2, we know that

$$\begin{aligned} \|\Theta^b f\|_{L^p} &\le C\|M^{\#}_{\mathfrak{L}}(\Theta^b f)\|_{L^p} \le C\sum_{j=1}^m \|b\|_{\Lambda^\beta_{\mathfrak{L}}}\|M_{s\tau,n\beta}(T_{j,2}f)\|_{L^p} \\ &\le C\sum_{j=1}^m \|b\|_{\Lambda^\beta_{\mathfrak{L}}}\|T_{j,2}f\|_{L^r} \le C\sum_{j=1}^m \|b\|_{\Lambda^\beta_{\mathfrak{L}}}\|f\|_{L^r}. \end{aligned}$$

Thus we complete the proof of Theorems 2.1–2.1.

4 Proof of Theorem 2.3

It is obvious that $\Lambda^0_{\mathfrak{L}}=\mathrm{BMO}_{\mathfrak{L}}$. Thus from the proof of Theorem 2.1, we have

$$M^{\#}_{\mathfrak{L}}(\Theta^b f)(x) \le C\sum_{j=1}^m \|b\|_{\mathrm{BMO}_{\mathfrak{L}}} M_{s\tau}(T_{j,2}f)(x).$$

Since $M_{s\tau}$ is boundedness on $L^p(\mathbb{R}^n)$, then

$$\|\Theta^b f\|_{L^p} \le C\|M^{\#}_{\mathfrak{L}}(\Theta^b f)\|_{L^p} \le C\sum_{j=1}^m \|b\|_{\mathrm{BMO}_{\mathfrak{L}}}\|M_{s\tau}(T_{j,2}f)\|_{L^p} \le C\sum_{j=1}^m \|b\|_{\mathrm{BMO}_{\mathfrak{L}}}\|f\|_{L^p}.$$

Therefore, we complete the proof of Theorem 2.3.

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由Campanato 型函数和与薛定谔算子相关的Riesz变换生成的Toeplitz算子的有界性

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摘 要: 本文研究了由 Campanato 型函数及与 Schrödinger 算子相关的 Riesz 变换生成的 Toeplitz 算子的有界性. 利用 Sharp 极大函数估计得到了 Toeplitz 算子 Θ^b 在 Lebesgue空间的有界性, 拓广了已有交换子的结果.

关键词: 交换子; Campanato 型函数; Riesz 变换; Schrödinger 算子

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