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一类含有小迟滞的奇摄动方程组的渐近解

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摘 要: 本文研究了一类含有小迟滞的奇摄动方程组的渐近解. 利用原问题的退化形式和伸长变量, 依据边界层特有的性质, 获得了边界层的渐近解. 推广了奇摄动方程组初值问题和小迟滞问题的研究结果.

关键词: 奇摄动; 方程组; 小延迟

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1 引言

奇摄动问题是国际数学界研究的热点问题, 奇摄动方程组渐近解与小迟滞问题渐近解都是热点问题之一, 如 O'malley[1] 研究的方程组的初值问题、方程组的边值问题以及具有一个小参数的微分 - 差分方程的解, 陈育森研究的双参数非线性积分微分方程组奇摄动边值问题 [2], 唐荣荣研究的一类非线性方程组的奇摄动问题 [3], 吴钦宽研究的伴有边界摄动非线性积分微分方程系统的奇摄动 [4], 欧阳成研究的一类非线性方程组的奇摄动初值问题 [5] 和有一个参数的小延迟的微分 - 差分方程渐近解 [6], 莫嘉琪研究的一个参数的奇摄动时滞反应扩散方程渐近解 [7]. 本文把方程组和迟滞问题同时考虑, 研究含有小迟滞的奇摄动方程组初值问题的渐近解.

问题

$$\frac{dx}{dt} = f(x, x'(t-\varepsilon), y'(t-\varepsilon)), \tag{1.1}$$

$$\varepsilon\frac{dy}{dt} = g(x, x'(t-\varepsilon), y'(t-\varepsilon)), \tag{1.2}$$

$$x(t) = \phi(t), -\varepsilon \le t \le 0, \tag{1.3}$$

$$y(t) = \psi(t), -\varepsilon \le t \le 0, \tag{1.4}$$

其中 $0 < \varepsilon \ll 1$.

为了叙述方便, 现作如下假设:

$[\mathrm{H}_1]$ (1.1)–(1.4) 的退化问题

$$\begin{aligned} &X_0'(t) = f(X_0(t), X_0'(t), Y_0'(t)), t \ge 0, \\ &g(X_0(t), X_0'(t), Y_0'(t)) = 0, t \ge 0, \\ &X_0(0) = \phi(0), Y_0(0) = \psi(0) \end{aligned}$$

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在某个区间 $0 \le t \le T$ 上存在唯一的连续可微解 $X_0(t), Y_0(t)$;

[H$_2$] f, g 在所考虑的区域内关于其变元无限可微;

[H$_3$] 存在正常数 $k > 0$, 有

$$| f_2'(X_0(0), \phi'(0), \psi'(0)) | < e^{-k} < 1, | f_3'(X_0(0), \phi'(0), \psi'(0)) | < e^{-k} < 1;$$

当 $| y | < | X_0'(0) | + | X_0'(0) - \phi'(0) |$, 有

$$| f_2'(X_0(0), y, \psi'(0)) | < e^{-k} < 1;$$

当 $| z | < | Y_0'(0) | + | Y_0'(0) - \psi'(0) |$, 有

$$| f_3'(X_0(0), \phi'(0), z) | < e^{-k} < 1;$$

[H$_4$] $f_3'(X_0(t), X_0'(t), Y_0'(t)) \neq 0$, $1 - f_2'(X_0(t), X_0'(t),$

$$Y_0'(t)) + \frac{f_3'(X_0(t), X_0'(t), Y_0'(t)) g_2'(X_0(t), X_0'(t), Y_0'(t))}{g_3'(X_0(t), X_0'(t), Y_0'(t))} \neq 0.$$

2 主要结果

设问题 (1.1)–(1.4) 的解为

$$\begin{aligned} x(t) &= X(t) + \varepsilon U(\tau) = \sum_{i=0}^{\infty} X_i(t)\varepsilon^i + \varepsilon \sum_{i=0}^{\infty} u_i(\tau)\varepsilon^i; \\ y(t) &= Y(t) + \varepsilon V(\tau) = \sum_{i=0}^{\infty} Y_i(t)\varepsilon^i + \varepsilon \sum_{i=0}^{\infty} v_i(\tau)\varepsilon^i, \end{aligned} \tag{2.1}$$

其中 $\tau = \frac{t}{\varepsilon}$, 问题 (1.1)–(1.4) 在远离 $t = 0$ 且 $t > 0$ 处外部解为

$$X(t) = \sum_{i=0}^{\infty} X_i(t)\mu^i, Y(t) = \sum_{i=0}^{\infty} Y_i(t)\varepsilon^i, \tag{2.2}$$

在 $t = 0$ 处 $x(t), y(t)$ 边界层校正项分别为

$$U(\tau) = \sum_{i=0}^{\infty} u_i(\tau)\varepsilon^i, V(\tau) = \sum_{i=0}^{\infty} v_i(\tau)\varepsilon^i.$$

把 (2.2) 式代入 (1.1), (1.2) 式得

$$\sum_{i=0}^{\infty} X_i'(t)\varepsilon^i = f(\sum_{i=0}^{\infty} \varepsilon^i X_i(t), \sum_{i=0}^{\infty} \varepsilon^i X_i'(t-\varepsilon), \sum_{i=0}^{\infty} \varepsilon^i Y_i'(t-\varepsilon)), t \ge 0, \tag{2.3}$$

$$\varepsilon \sum_{i=0}^{\infty} Y_i'(t)\varepsilon^i = g(\sum_{i=0}^{\infty} \varepsilon^i X_i(t), \sum_{i=0}^{\infty} \varepsilon^i X_i'(t-\varepsilon), \sum_{i=0}^{\infty} \varepsilon^i Y_i'(t-\varepsilon)), t \ge 0, \tag{2.4}$$

所以

$$
\begin{aligned}
&X_i'(t)=f_1'(X_0(t),X_0'(t),Y_0'(t))X_i(t)+f_2'(X_0(t),X_0'(t),Y_0'(t))X_i'(t)\\
&+f_3'(X_0(t),X_0'(t),Y_0'(t))Y_i'(t)+A_i(t),t\geq 0, \qquad (2.5)\\
&g_1'(X_0(t),X_0(t),Y_0'(t))X_i(t)+g_2'(X_0(t),X_0'(t),Y_0'(t))X_i'(t)\\
&+g_3'(X_0(t),X_0'(t),Y_0'(t))Y_i'(t)+B_i(t)=0,t\geq 0, \qquad (2.6)
\end{aligned}
$$

这里 $i=1,2,3,\cdots$, 且 $A_i(t),B_i(t)$ 是逐次确定的光滑函数. 把 (2.1) 式代入 (1.3), (1.4) 式, 得

$$X_i(0)=u_{i-1}(0),Y_i(0)=v_{i-1}(0),i=1,2,3,\cdots. \qquad (2.7)$$

下面求 $u_i(0),v_i(0)$ $(i=0,1,2,\cdots)$. 把 (2.1) 式代入 (1.1), (1.2) 式得

$$
\begin{aligned}
&\sum_{i=0}^{\infty}\dot{u}_i(\tau)\varepsilon^i=f(\sum_{i=0}^{\infty}\varepsilon^iX_i(t)+\varepsilon\sum_{i=0}^{\infty}u_i(\tau)\varepsilon^i,\sum_{i=0}^{\infty}\varepsilon^iX_i'(t-\varepsilon)+\sum_{i=0}^{\infty}\dot{u}_i(\tau-1)\varepsilon^i,\\
&\sum_{i=0}^{\infty}\varepsilon^iY_i'(t-\varepsilon)+\sum_{i=0}^{\infty}\varepsilon^i\dot{v}_i(\tau-1))-f(\sum_{i=0}^{\infty}\varepsilon^iX_i(t),\sum_{i=0}^{\infty}\varepsilon^iX_i'(t-\varepsilon),\sum_{i=0}^{\infty}\varepsilon^iY_i'(t-\varepsilon)),\\
&t\geq 0,\\
&\sum_{i=0}^{\infty}\dot{v}_i(\tau)\varepsilon^i=g(\sum_{i=0}^{\infty}\varepsilon^iX_i(t)+\varepsilon\sum_{i=0}^{\infty}u_i(\tau)\varepsilon^i,\sum_{i=0}^{\infty}\varepsilon^iX_i'(t-\varepsilon)+\sum_{i=0}^{\infty}\dot{u}_i(\tau-1)\varepsilon^i,\\
&\sum_{i=0}^{\infty}\varepsilon^iY_i'(t-\varepsilon)+\sum_{i=0}^{\infty}\varepsilon^i\dot{v}_i(\tau-1))-g(\sum_{i=0}^{\infty}\varepsilon^iX_i(t),\sum_{i=0}^{\infty}\varepsilon^iX_i'(t-\varepsilon),\sum_{i=0}^{\infty}\varepsilon^iY_i'(t-\varepsilon)),\\
&t\geq 0.
\end{aligned}
$$

当 $0<\tau\leq 1$ 时,

$$
\begin{aligned}
\dot{u}_0(\tau)&=f(X_0(0),\phi'(0),\psi'(0))-f(X_0(0),X_0'(0),Y_0'(0)),\\
\dot{v}_0(\tau)&=g(X_0(0),\phi'(0),\psi'(0))-g(X_0(0),X_0'(0),Y_0'(0));
\end{aligned}
$$

当 $\tau\geq 1$ 时,

$$\dot{u}_0(\tau)=f(X_0(0),X'(0)+\dot{u}_0(\tau-1),Y'(0)+\dot{v}_0(\tau-1))-f(X_0(0),X_0'(0),Y_0'(0)),$$

$$\dot{v}_0(\tau)=g(X_0(0),X'(0)+\dot{u}_0(\tau-1),Y'(0)+\dot{v}_0(\tau-1))-g(X_0(0),X_0'(0),Y_0'(0)).$$

所以 $\dot{u}_0(\tau),\dot{v}_0(\tau)$ 为分段常数. 对于非负整数 $p,p\leq\tau\leq p+1$. 记

$$\dot{u}_0(\tau)=M_p^0,\dot{v}_0(\tau)=N_p^0,$$

所以

$$u_0(\tau)=u_0(0)+\int_0^{\tau}\dot{u}_0(s)ds=u_0(0)+\sum_{j=0}^{p-1}M_j^0+(\tau-p)M_p^0,$$

$$v_0(\tau)=v_0(0)+\int_0^\tau \dot{v}_0(s)ds=v_0(0)+\sum_{j=0}^{p-1}N_j^0+(\tau-p)N_p^0.$$

因为当 $\tau\to\infty$ 时, $u_0(\tau)\to 0, v_0(\tau)\to 0$, 所以

$$u_0(0)=-\sum_{j=0}^{\infty}M_j^0, v_0(0)=-\sum_{j=0}^{\infty}N_j^0.$$

同时

$$\dot{u}_i(\tau)=M_p^i(\tau),\dot{v}_i(\tau)=N_p^i(\tau)\quad(p\le\tau\le p+1,\quad i=1,2,\cdots),$$

这里 $M_p^i(\tau),N_p^i(\tau)$ 是依次确定的函数, 并且当 τ 取整数值时, $\dot{u}_i(\tau),\dot{v}_i(\tau)$ 一般是间断的. 所以

$$u_i(\tau)=u_i(0)+\sum_{l=0}^{p-1}(\int_l^{l+1}M_l^i(s)ds)+\int_p^\tau M_p^i(s)ds\quad(p\le\tau\le p+1),$$

$$v_i(\tau)=v_i(0)+\sum_{l=0}^{p-1}(\int_l^{l+1}N_l^i(s)ds)+\int_p^\tau N_p^i(s)ds\quad(p\le\tau\le p+1).$$

当 $\tau\to\infty$ 时, $u_i(\tau)\to 0, v_i(\tau)\to 0$, 所以

$$u_i(0)=-\sum_{l=0}^{\infty}\int_l^{l+1}M_l^i(s)ds,\quad v_i(0)=-\sum_{l=0}^{\infty}\int_l^{l+1}N_l^i(s)ds.$$

又因为 [H$_3$], $\dot{u}_i(\tau),\dot{v}_i(\tau)$ $(i=0,1,2\cdots)$ 都是指数性小项, 且

$$\lim_{\tau\to\infty}u_i(\tau)=0,\lim_{\tau\to\infty}v_i(\tau)=0,$$

所以 $u_i(\tau),v_i(\tau)$ $(i=0,1,2\cdots)$ 都是指数性小项, 且 $v_i(0)$ $(i=0,1,2\cdots)$ 是有限值 [1]. 这样就可以利用 (2.5)–(2.7) 式与 [H$_4$] 依次得 $X_i(t),Y_i(t)$ $(i=1,2,\cdots)$. 因此原方程的内部解、外部解都可依次解出, 从而得到原方程的渐近解.

定理 在 [H$_1$]–[H$_4$] 下, 在 $0\le t\le T$ 上, 问题 (1.1)–(1.4) 有唯一解

$$x(t)=X(t)+\varepsilon U(\tau)=\sum_{i=0}^{\infty}X_i(t)\varepsilon^i+\varepsilon\sum_{i=0}^{\infty}u_i(\tau)\varepsilon^i,$$

$$y(t)=Y(t)+\varepsilon V(\tau)=\sum_{i=0}^{\infty}Y_i(t)\varepsilon^i+\varepsilon\sum_{i=0}^{\infty}v_i(\tau)\varepsilon^i,$$

其中 $\tau=\frac{t}{\varepsilon}$.

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THE ASYMPTOTIC SOLUTION OF A CLASS OF SINGULARLY PERTURBED SYSTEM OF EQUATIONS WITH SMALL DELAY

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Abstract: The asymptotic solution of a class of singularly perturbed system of equations with small delay are studied in this paper. By the degenerate form of the original problem, stretched variable and the nature of the boundary layers, the asymptotic solutions of the boundary layers are given, which generalizes the findings of singularly perturbed system of equations and small delay problems.

Keywords: singular perturbation; system of equations; small delay

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