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Asymptotic Normality of the Empirical Distribution under Negatively Associated Sequences and its Applications

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Abstract: By the well-known large and small blocks parting method for dependent situations, we establish the asymptotic normality of the Empirical Distribution Function under Negatively Associated Sequences. As its application in reliablity problems, a natural estimate $\overline{F}_n(x)$ for the survival function $\overline{F}(x) = P(X > x)$ is proposed, and the asymptotic normality of $n^{\frac{1}{2}}[\overline{F}_n(x) - \overline{F}(x)]$ is established.

Key words: NA sequences; empirical distribution; survival function; asymptotic normality.
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1. Introduction

Random variables $\{X_1, \cdots, X_n; n \geq 2\}$ are said to be negatively associated (NA), if for every pair of disjoint subsets A_1 and A_2 of $\{1, 2, \cdots, n\}$,

$$\mathrm{Cov}[g_1(X_i, i \in A_1), g_2(X_j, j \in A_2)] \leq 0,$$

where g_1 and g_2 are increasing for every variable (or are decreasing for every variable), such that covariance exists. Random variables sequence $\{X_j; j \in N\}$ are said to be negatively associated if every subfamily is negatively associated. This definition was introduced by Joag-Dev and Proschan[1]. Because of its wide application in reliability theory problems, statistical mechanics, probability/stochastic processes and statistics, the notion of NA random variables have received more and more attention recently.

Let $X_1, X_2, \cdots, X_n$ be a strictly stationary negatively associated random variable sequence with distribution (d.f.)F, and set $\overline{F}(x) = P(X > x)$ for the survial function, where X is distributed according to F; Of course, $\overline{F}(x) = 1 - F(x)$. The estimate $F_n(x)$ is the empirical d.f. based on $X_1, X_2, \cdots, X_n$, and $\overline{F}_n(x) = 1 - F_n(x)$.

The estimation of the survival function $\overline{F}(x)$ and the establishment of asymptotically optimal properties of the proposed estimate $\overline{F}_n(x)$ is, clearly, of interest on its own right. It is also essential in estimating the hazard rate or failure rate at x, $r(x)$, where $r(x) = \frac{f(x)}{\overline{F}(x)}$, $x \in R$, with

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$\overline{F}(x) > 0$ and f is the probability density function (p.d.f.) of F. Roussas[2] established asymptotic normality for $n^{\frac{1}{2}}[\overline{F}_n(x) - \overline{F}(x)]$ under dependence conditions. Yuan Ming and Su Chun[3] developed the weak convergence for empirical process based on NA. Motivated by them, in the note, we shall established the asymptotic normality of $n^{\frac{1}{2}}[F_n(x) - F(x)]$ under NA using large and small blocks, under weaker assumptions than the references. As its application in reliablity problems, a natural estimate $\overline{F}_n(x)$ for the survival function $\overline{F}(x) = P(X > x)$ is proposed, and the asymptotic normality of $n^{\frac{1}{2}}[\overline{F}_n(x) - \overline{F}(x)]$ is also established.

The paper is organized as follows. The required assnmptions and notation are introduced in Section 2. The main result is also stated in the same section, but the proof is deferred to Section 4. Section 3 is devoted to the establishment of some auxliary results.

2. Assumptions, notation and main result

In this section, the assumptions used throughout the paper are gathered together for easy reference, the necessary notation is introduced, and the main result Theerem 2.1 is stated.

Assumptions:

(A1) (i) The random variables $X_1, X_2, \cdots$ constitute a strictly stationary sequence and have distribution function and probability density function F and f, respectively.

(ii) $X_1, X_2, \cdots$ are NA.

(iii) The joint distribution of X_1, X_j is $F_{1,j+1}(x, y)$, $\forall x, y, \in R$ and $j \geq 1$.

Let $u(n) = \sum\limits_{j=n}^{\infty} \sup\limits_{(x,y)\in R} |F_{1,j+1}(x, y) - F(x)F(y)|$. Thus, $u(1) < \infty$.

(iv) The probability density function is bounded.

(A2)(i) Let $0 < \alpha = \alpha_n < n,\ 0 < \beta = \beta_n < n$ be integers tending to ∞ along with n.

(ii) Let $0 < \mu = \mu_n \stackrel{n\to\infty}{\longrightarrow} \infty$ be defined by $\mu = [n/(\alpha + \beta)]$ ($[x]$ stands for the integral part of x).

(iii) $\mu(\alpha + \beta) \leq n$, $\mu(\alpha + \beta)/n \to 1$ and $\mu\beta/n \to 0$.

(iv) $\frac{\alpha^2}{n} \to 0$.

On the basis of $X_1, X_2, \cdots, X_n$, define $F_n(x)$ by

$$F_n(x) = \frac{1}{n}\sum_{j=1}^{n} Y_j(x), \quad Y_j(x) = I_{(X_j \leq x)}. \tag{2.1}$$

Thus

$$EY_j(x) = F(x), \quad \mathrm{Var}[Y_j(x)] = F(x)[1 - F(x)] \hat{=} \sigma_1^2(x), \tag{2.2}$$

and

$$n^{\frac{1}{2}}[F_n(x) - F(x)] = n^{-\frac{1}{2}}\sum_{j=1}^{n} Z_j, \quad Z_j = Y_j(x) - EY_j(x)), \tag{2.3}$$

where Z_j stands for $Z_j(x)$. The derivations below will use the fact that random variables $Z_j, j = 1, \cdots, n$, are bounded by 1. We split the frist expression in (2.3) as follows:

$$n^{-\frac{1}{2}}\sum_{j=1}^{n} Z_j = n^{-\frac{1}{2}}(S_n + T_n + T_n'), \tag{2.4}$$

where

$$S_n = \sum_{m=1}^{\mu} y_m, \quad T_n = \sum_{m=1}^{\mu} y'_m, \quad T'_n = y'_{\mu+1}, \tag{2.5}$$

and

$$y_m = \sum_{i=k_m}^{k_m+\alpha-1} Z_i, \;\; k_m = (m-1)(\alpha+\beta)+1, m = 1, \cdots, \mu; \tag{2.6}$$

$$y'_m = \sum_{i=l_m}^{l_m+\beta-1} Z_j, l_m = (m-1)(\alpha+\beta)+\alpha+1, m = 1, \cdots, \mu; \tag{2.7}$$

$$y'_{\mu+1} = \sum_{j=\mu(\alpha+\beta)+1}^{n} Z_j. \tag{2.8}$$

Let

$$\sigma_0^2 = \sigma_1^2(x) + 2\sigma_2(x)(\geq 0), \text{ where } \; \sigma_1^2(x) = F(x)[1-F(x)] \text{ and } \sigma_2(x) = \sum_{j=1}^{\infty} E(Z_1 Z_{j+1}). \tag{2.9}$$

Theorem 2.1 *Assume A1 and A2, then*

$$n^{\frac{1}{2}}[F_n(x) - F(x)] \xrightarrow{d} N(0, \sigma_0^2). \tag{2.10}$$

Corollary 2.2 *Assume A1 and A2, then*

$$n^{\frac{1}{2}}[\overline{F}_n(x) - \overline{F}(x)] \xrightarrow{d} N(0, \sigma_0^2), \tag{2.11}$$

where " $\xrightarrow{d}$ " denotes the convergence in distribution.

3. Some auxiliary Results

In this section, a number of lemmas are presented and will be used in the subsequent parts of the paper.

Observe that

$$|\mathrm{Cov}(Z_i, Z_j)| = |\mathrm{Cov}(Y_i, Y_j)| = |\mathrm{Cov}(I_{(X_i \leq x)}, \mathrm{Cov}(I_{(X_j \leq y)})| = |F_{i,j+1}(x,y) - F(x)F(y)|. \tag{3.1}$$

Lemma 3.1 *Assume A1. For any integer $k \geq 2$, we have*

$$|\sum_{1\leq i<j\leq k} E(Z_i Z_j)| \leq Ck < \infty.$$

Proof We note that

$$\begin{aligned} &|\sum_{1\leq i<j\leq k} \mathrm{Cov}(Z_i, Z_j)| \leq \sum_{1\leq i<j\leq k} |\mathrm{Cov}(Z_i, Z_j)| \\ &\leq C \sum_{1\leq i<j\leq k} |F_{i,j}(x,y) - F(x)F(y)| \leq C \sum_{i=1}^{k} \sum_{j=1}^{\infty} |F_{1,j+1}(x,y) - F(x)F(y)| \leq Ckc(1) \\ &= Ck < \infty. \end{aligned}$$

Lemma 3.2 *Under (A1) and (A2),*

$$\frac{\mu}{n}E(y_i')^2 \to 0.$$

Proof By stationarity, it is easy to see that

$$\begin{aligned}
\frac{\mu}{n}E(y_i')^2 &= \frac{\mu}{n}E(\sum_{j=\beta+1}^{\alpha+\beta} Z_j)^2 = \frac{\mu}{n}E(\sum_{j=1}^{\beta} Z_j)^2 \\
&= \frac{\beta\mu}{n}\sigma_1^2(x) + 2\frac{\mu}{n}\sum_{j=1}^{\beta-1}(\beta-j)\mathrm{Cov}(Z_1, Z_{j+1}) \\
&\le \frac{\beta\mu}{n}\sigma_1^2(x) + 2\frac{\mu}{n}\sum_{j=1}^{\beta-1}(\beta-j)|F_{1,j+1}(x,y) - F(x)F(y)| \\
&\le \frac{\beta\mu}{n}\sigma_1^2(x) + 2(\frac{\mu}{n}\beta)\sum_{j=1}^{\infty}|F_{1,j+1}(x,y) - F(x)F(y)| \to 0.
\end{aligned}$$

Lemma 3.3 *Under (A1) and (A2),*

$$\frac{1}{n}\sum_{1\le i<j\le \mu} E(y_i'y_j') \to 0.$$

Proof Calculate the expectation $E(y_1'y_{j+1}')$, replace y_1' and y_{j+1}' by their expressions in (2.7), and employ the stationarity to obtain

$$\begin{aligned}
|E(y_1'y_{j+1}')| &= |\sum_{i=\beta+1}^{\alpha+\beta}\ \sum_{l=j(\alpha+\beta)+\beta+1}^{(j+1)(\alpha+\beta)} \mathrm{Cov}(Z_i, Z_l)| \\
&= |\sum_{r=1}^{\beta}(\beta-r+1)\mathrm{Cov}(Z_1, Z_{j(\alpha+\beta)+r})| + |\sum_{r=1}^{\beta-1}(\beta-r)\mathrm{Cov}(Z_{r+1}, Z_{j(\alpha+\beta)+1})| \\
&= |\sum_{r=1}^{\beta}(\beta-r+1)\mathrm{Cov}(Z_1, Z_{j(\alpha+\beta)+r})| + |\sum_{r=1}^{\beta-1}(\beta-r)\mathrm{Cov}(Z_1, Z_{j(\alpha+\beta)-r+1})| \\
&\le \beta|\sum_{r=j(\alpha+\beta)-(\beta-2)}^{j(\alpha+\beta)+\beta} \mathrm{Cov}(Z_1, Z_r)| = \beta|\sum_{r=j(\alpha+\beta)-(\beta-1)}^{j(\alpha+\beta)+(\beta-1)} \mathrm{Cov}(Z_1, Z_{r+1})| \\
&\le \beta\sum_{r=j(\alpha+\beta)-(\beta-1)}^{j(\alpha+\beta)+(\beta-1)} |F_{1,r+1}(x,y) - F(x)F(y)|. \qquad (3.2)
\end{aligned}$$

Then, by (3.2), we have

$$\begin{aligned}
|\frac{1}{n}\sum_{1\le i<j\le\mu} E(y_1'y_j')| &\le \frac{1}{n}\sum_{j=1}^{\mu-1}(\mu-j)|E(y_1'y_{j+1}')| \\
&\le \frac{\mu}{n}\sum_{j=1}^{\mu-1}|E(y_1'y_{j+1}')|
\end{aligned}$$

$$\leq C\frac{\beta\mu}{n}\sum_{j=1}^{\mu}\sum_{r=j(\alpha+\beta)-(\beta-1)}^{j(\alpha+\beta)+(\beta-1)}|F_{1,r+1}(x,y)-F(x)F(y)|$$

$$= C\frac{\beta\mu}{n}\sum_{r=\alpha+1}^{\mu(\alpha+\beta-1)}|F_{1,r+1}(x,y)-F(x)F(y)| = C(\frac{\beta\mu}{n})u(\alpha+1)\to 0.$$

Lemma 3.4 *Under (A1) and (A2), with T_n, T_n' given by (2.5), the following conclusions hold*

(i). $\frac{1}{n}ET_n^2\to 0$; *(ii).* $\frac{1}{n}E(T_n')^2\to 0$.

Proof (i). By Lemmas 3.2 and 3.3 , we have

$$\frac{1}{n}ET_n^2 = \frac{\mu}{n}E(y_1')^2+\frac{2}{n}\sum_{1\leq i<j\leq\mu}E(y_i'y_j')\to 0.$$

(ii). Observing the double inequality $\mu(\alpha+\beta)\leq n<(\mu+1)(\alpha+\beta)$, we have

$$\frac{n-\mu(\alpha+\beta)}{n}<\frac{1}{\mu}.$$

Using the stationarity and applying Lemma 3.3 for $k=n-\mu(\alpha+\beta)$, we have

$$\frac{1}{n}E(T_n')^2 = \frac{n-\mu(\alpha+\beta)}{n}\sigma_1^2(x)+\frac{2}{n}\sum_{\mu(\alpha+\beta)+1\leq i<j\leq n}\mathrm{Cov}(Z_j,Z_j)$$

$$\leq \frac{n-\mu(\alpha+\beta)}{n}\sigma_1^2(x)+C\frac{2(n-\mu(\alpha+\beta))}{n}\leq\frac{1}{\mu}[\sigma_1^2(x)+C]\to 0.$$

Lemma 3.5 *Under (A1) and (A2) and for any subaequence $\{m\}$ of $\{n\}$ tending to infinity,*

$$\lim_{m\to\infty}\frac{1}{m}\sum_{1\leq i<j\leq m}E(Z_iZ_j)=\lim_{n\to\infty}\frac{1}{m}\sum_{j=1}^{m}E(Z_1Z_{j+1})=\sigma_2(x)$$

for some finite $\sigma_2(x)$. And then $\frac{1}{\alpha}Ey_1^2\to\sigma_1^2(x)+2\sigma_2(x)=\sigma_0^2(x)$.

Proof It is easy to see that

$$\frac{1}{m}\sum_{1\leq<j\leq m}E(Z_iZ_j)=\frac{1}{m}\sum_{j=1}^{m-1}(m-j)E(Z_1Z_{j+1})$$

$$=\sum_{j=1}^{m-1}E(Z_1Z_{j+1})-\frac{1}{m}\sum_{j=1}^{m-1}jE(Z_1Z_{j+1})$$

$$=\sum_{j=1}^{m-1}E(Z_1Z_{j+1})-\frac{1}{m}\sum_{j=1}^{m}jE(Z_1Z_{j+1})+E(Z_1Z_{m+1}).$$

But $|E(Z_1Z_{m+1})|\leq C|F_{1,m+1}(x,x)-F^2(x)|\to 0$, (as $m\to\infty$), then

$$|\sum_{j=1}^{\infty}E(Z_1Z_{j+1})|\leq\sum_{j=1}^{\infty}|E(Z_1Z_{j+1})|\leq C\sum_{j=1}^{\infty}|F_{1,j+1}(x,y)-F(x)F(y)|=Cu(1)<\infty,$$

so that $\sum_{j=1}^{\infty} E(Z_1 Z_{j+1})$ converges to a finite limit. Then by the Kronecker Lemma, we obtain

$$\frac{1}{m}\sum_{j=1}^{n} jE(Z_1 Z_{j+1}) \to 0.$$

And

$$\lim_{n\to\infty}\frac{1}{m}\sum_{1\le i<j\le m} E(Z_i Z_j) = \lim_{n\to\infty}\sum_{j=1}^{m} E(Z_1 Z_j) = \sigma_2(x).$$

The second assertion is an immediate consequence of the first by taking $\{m\} = \{\alpha\}$.

Corollary 3.6 *Under (A1) and (A2), it holds that*

$$\frac{\mu}{n} E y_1^2 \to \sigma_0^2(x).$$

Proof This is so because $\frac{\beta\mu}{n} \to 0$ implies $\frac{\alpha\mu}{n} \to 1$ and

$$\frac{\mu}{n} E y_1^2 = (\frac{\alpha\mu}{n})\frac{1}{\alpha} E y_1^2 \to \sigma_0^2(x). \quad \text{(by Lemma 3.5)}$$

4. Asymptotic normality $F_n(x)$

In this section, we shall establish (2.10). This is done in two steps. Frist, it is shown that the characteristic functions of $\sum\limits_{m=1}^{\mu} n^{-\frac{1}{2}} y_m$, minus the product of the characteristic functions of $n^{-\frac{1}{2}} y_m, m = 1, 2, \cdots, \mu$, coverge to 0 in absolute value. And secondly, it is proved that the distribution determined by the product of the characteristic functions of $\sum\limits_{m=1}^{\mu} n^{-\frac{1}{2}} y_m$ is asymptotically the anticipated distribution $N(0, \sigma_0^2)$. The relevant details are in Lemma 4.2 and the proof of the relation (4.1).

Lemma 4.1[5] *Let $\{X_i, 1 \le i \le n\}$ be NA, then*

$$|E\exp(it\sum_{j=1}^{n} X_j) - \prod_{j=1}^{n} E\exp(itX_j)| \le 4t^2 \sum_{1\le i<j\le n} \mathrm{cov}(X_i, X_j).$$

Lemma 4.2 *Under (A1) and (A2), we have*

$$|E\exp(it\sum_{m=1}^{\mu} n^{-\frac{1}{2}} y_m) - \prod_{m=1}^{\mu} E\exp(itn^{-\frac{1}{2}} y_m)| \to 0$$

Proof By $\frac{\mu\alpha}{n} \to 0$ and the stationarity, we have

$$|E\exp(it\sum_{m=1}^{\mu} n^{-\frac{1}{2}} y_m) - \prod_{m=1}^{\mu} E\exp(itn^{-\frac{1}{2}} y_m)|$$

$$\le 4t^2 \sum_{1\le i<j\le \mu} \sum_{s=k_i}^{k_i+\alpha-1} \sum_{l=k_j}^{k_j+\alpha-1} |\mathrm{Cov}(Z_s, Z_l)|$$

$$\le C\frac{4t^2}{n}\sum_{1\le i<j\le\mu}\sum_{s=k_i}^{k_i+\alpha-1}\sum_{l=k_j}^{k_j+\alpha-1}|F_{s,l}(x,y)-F(x)F(y)|$$

$$\le C\frac{4t^2}{n}\sum_{i=1}^{\mu-1}\sum_{s=k_i}^{k_i+\alpha-1}\sum_{j=i+1}^{\mu}\sum_{l=k_j}^{k_j+\alpha-1}|F_{s,l}(x,y)-F(x)F(y)|$$

$$= C\frac{4t^2}{n}\sum_{i=1}^{\mu-1}\sum_{s=k_i}^{k_i+\alpha-1}\sum_{t=\beta}^{\infty}|F_{s,l}(x,y)-F(x)F(y)|$$

$$\le Ct^2\frac{\mu\alpha}{n}u(\beta)\to 0.$$

Lemma 4.3 *Assume (A1) and (A2), then*

$$n^{-\frac{1}{2}}S_n \xrightarrow{d} N(0,\sigma_0^2).$$

Proof Consider the r.v.s $n^{-1/2}y_m, m=1,2,\cdots,\mu$ and let $z_{nm}, m=1,2,\cdots,\mu$ be independent r.v.s with distribution of $n^{-1/2}y_1$, so that $Ez_{nm}=0$. by Lemma 4.2, we have

$$|E\exp(it\sum_{m=1}^{\mu}n^{-\frac{1}{2}}y_m)-\prod_{m=1}^{\mu}E\exp(itz_{nm})|\to 0$$

or by the independence of the z_{nm}

$$|E\exp(it\sum_{m=1}^{\mu}n^{-\frac{1}{2}}y_m)-E\exp(it\sum_{m=1}^{\mu}z_{nm})|\to 0.$$

Now we prove that

$$\sum_{m=1}^{\mu}z_{nm}\xrightarrow{d}N(0,\sigma_0^2(x)). \tag{4.1}$$

Set

$$s_n=\sum_{m=1}^{\mu}\mathrm{Var}(z_{nm}),\quad Z_{nm}=\frac{z_{nm}}{s_n}.$$

Then the r.v.s $Z_{nm}, m=1,2,\cdots,\mu$ are indepenent identically distributed with $EZ_{n1}=0$, $Var(Z_{n1})=\frac{1}{\mu}$, so that $\sum\limits_{m=1}^{\mu}\mathrm{Var}(Z_{nm})=1$. By Corrollary 3.1, we have $s_n^2\to\sigma_0^2(x)$. Then the convergence (4.1) is equivalent to

$$\sum_{m=1}^{\mu}Z_{nm}\xrightarrow{d}N(0,1). \qquad (4.2).$$

Let G_n be the d.f. of $\frac{n^{-1/2}y_1}{s_n}$. Then for every $\varepsilon>0$,

$$g_n(\varepsilon)=\mu\cdot\int_{(|x|\ge\varepsilon)}x^2\mathrm{d}G_n\to 0.$$

From (2.4), $|\frac{n^{-1/2}y_1}{s_n}| \le (C\alpha)/(s_n\sqrt{n})$, where C is a bound for K, so that $|\frac{n^{-1/2}Z_{n1}}{s_n}| \le (C\alpha)/(s_n\sqrt{n})$. Thus

$$\begin{aligned} g_n(\varepsilon) &= \mu \cdot \int_{(|x|\ge\varepsilon)} x^2 \mathrm{d}G_n \\ &\le \mu \cdot E[Z_{nm}^2 I_{(|Z_{n1}|\ge\varepsilon)}] \le \frac{C^2\alpha^2\mu}{ns_n^2} P(|Z_{n1}| \ge \varepsilon) \\ &\le \frac{C^2\alpha^2\mu}{ns_n^2} \cdot \mathrm{Var}(Z_{n1}) = \frac{C^2}{ns_n^2} \cdot \mu \mathrm{Var}(Z_{n1}) \cdot \frac{\alpha^2}{n} \to 0 \ \ (\text{where } \frac{\alpha^2}{n} \to 0). \end{aligned}$$

Then by the Feller-Lindeberg Criterion, (4.1) is proved. So the desired result follows.

Proof of Theorem 2.1 By Lemma 3.4, we have $n^{-1}E[T_n^2 + (T_n')^2] \to 0$, so $n^{-\frac{1}{2}}(T_n + T_n') \xrightarrow{P} 0$. In conjunction with Lemma 4.3, the desired result (2.10) yields.

Proof of Corollary 2.2 Observe that $\overline{F}_n(x) - \overline{F}(x) = F(x) - F_n(x)$. By Theorem 2.1, the desired result (2.11) is obtained.

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NA 序列下经验分布函数的渐近正态性及其应用

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摘要: 本文在 NA 负相协序列下利用熟知的相依情形的大小块分割的方法，建立了经验分布函数的渐近正态性. 作为在可靠性中的应用，得到了生存函数 $\overline{F}(x) = P(X > x)$ 估计的渐近正态性.

关键词: NA 列；经验分布函数；渐近正态性；生存函数.